Free Vibration Analysis of Multi-Body System

Dr. Bahaa I.K. **Dr. Riyah N.K.** **Husam M.A.**
University of Baghdad **University of Anbar**

Abstract
In this research a simply supported beam is used as a master structure with unknown number of attachments (fuzzy substructure) which is modeled as a system of 1-DOF attachments. Two types of attachments models were studied, namely 1-DOF mass attachment model and 1-DOF mass-spring attachment model. It is shown that the effect of attachments on the master structure natural frequencies when modeled as (mass-spring substructure) is larger than that when modeled as (mass substructure) for the same attachment mass. Engineering Statistics and normal distribution were used to find the values of the attachments to be added to the simply supported beam to improve the dynamical properties of the master structure and to find the best distribution of the attachment. The results also show that the distribution of the additional substructure can produce a great change in the natural frequencies so that the proposed statistical approach can be used to find the best distribution of attachments and number, value and location of the additional substructure .

Keywords: dynamic, structure analysis, multi-body, vibration, fuzzy

1.Introduction

A fuzzy structure is defined as a master structure that is accessible to conventional deterministic modeling, coupled with fuzzy substructures which are not accessible to conventional deterministic modeling and for which a probabilistic model is used. The part of the structure which can be modeled conventionally (shell, ribs, bulkheads, etc.) is called the master structure. The remaining part of the structure, which cannot be modeled conventionally, is called the fuzzy substructure as shown in **Fig.(1)**.

D. Moens et al. [1] showed that small uncertainties on models are known to cause possibly large uncertainties on analysis results. Therefore, techniques have been developed to include the effect of the uncertainties. On the model in the calculation of the results of an analysis, the Monte Carlo simulation calculates these uncertainties by performing a stochastic analysis on the results of a large number of samples of one calculation.

Christian Soize [2] showed that the structural complexity can be induced by secondary mechanical subsystems attached to the ``master'' structure or by ``local eigen modes'' of some continuum elastic sub elements of the master structure; these local eigen modes induce a structural complexity when the model of these sub elements can only restitute the elasto-static behavior but not its elasto-dynamic response. Within this context, a model is presented of the apparent vibration damping of the master structure due to the vibrations of the structural complexity. This vibration-damping model is deduced from the theory of fuzzy structures that was previously developed by the author. Presently, this model uses only the mean part of the probabilistic fuzzy law of the fuzzy substructure

Victor et al. [3] present the application of the fuzzy structures analysis (FSA) procedures of Soize[2] to prototypical aerospace panels in MSC/NASTRAN, a large commercial finite element program .A brief introduction to the FSA procedures is first provided. The implementation of the FSA methods is then disclosed, and the method is validated by comparison to published results for the forced vibrations of a fuzzy beam.

Bahaa & wedad [4] showed that the dynamic analysis (free vibration) of multi-body system flexibly supported by linear mountings. The multi-body supporting system can be represented by a cantilever plate structure which contains an additional bodies like electrical and mechanical parts .An important type of multi-body systems constructing from a cantilever plate which is used as a flexible-mounting base for other components. This method for fixing can commonly used for isolation of electronic components (or electronic cards). The results showed that the masses of the additional bodies and their distribution on the plate structure can produce a great change in the natural frequencies and the mode shapes of the mounting system .

A. V. Pesterev [5], showed that the problem of calculating the dynamic response of a one-dimensional distributed parameter system carrying multiple moving oscillators is examined. A solution procedure is suggested that reduces the problem to the integration of a system of linear ordinary differential equations governing the time-dependent coefficients of the series expansion of the response in terms of the eigen functions of the continuous structure. The program implementation of the solution procedure is discussed and numerical results are presented.

Jong-shyong [6] showed that since the literature relating to the natural frequencies and mode shapes of the double-tapered wedge beams carrying multiple point masses is rare .The object of this paper is to present some information in this aspect. First of all, the closed-form solutions in the terms of the Bessel functions for the natural frequencies and normal mode shape of the (bare) wedge beams (with out carrying any point masses) were determined. Next , the partial differential equation of motion for the (loading) wedge beams (carrying any number point masses) were transformed in to the matrix equation by using the expansion theorem and the foregoing natural frequencies and normal mode shape of the (bare) wedge beam . Finally, the eigen value equation associated with the last matrix equation was solved to give the natural frequencies and the mode shapes of the (loading) wedge beams.

Jia-Jang [7] showed that to determine the natural frequencies and mode shapes of beams they have to carry any number of two-degree-of-freedom (DOF) spring-mass systems by means of two finite element methods FEM1 and FEM2. FEM1 is the conventional finite element method (FEM), in which each two-DOF spring-mass system is considered as a finite element and then the assembly technique is used to establish the overall property matrices of the constrained beam. FEM2 is an alternative approach, in which each two-DOF spring-mass system is replaced by four effective springs and then the overall property matrices of the constrained beam are obtained by considering the whole structural system as the unconstrained beam elastically supported by the effective springs.

Tianjian et al.[8] show that the Human-structure interaction has not been considered before. One reason is that the human body is traditionally considered as an inert mass in structural vibration. It was found that the human body did not act as an inert mass but as a mass-spring-damper system. This finding, together with many site measurements, forms a basis of a new topic, human-structure interaction and leads to new applications .

The present work deals with studying the effect of one degree of freedom mass and mass-spring attachment models on natural frequencies and their associated mode shapes for free vibration of a simply supported beam . Also, the best distribution of the attachments according to the normal distribution criteria is sought.

2.Theoretical considerations:

2.1.Beam theory of vibration:

The bending equation for the beam shown in **Fig.(2)** is[9]:

$$\frac{d^2}{dx^2}\left[EI \frac{d^2 w}{dx^2}\right] + m\ddot{w} = P(x,t) \tag{1}$$

In a continuous system , there is an infinite number of natural frequencies (eigenvalues) and associated modes (eigenvectors) so that:

$$w(x,t) = \overline{w}(x) e^{i\omega t} \tag{2}$$

Consider the homogeneous case (P(x,t) = 0) and (EI) does not vary with x, thus:

$$EI \frac{d^4 w}{dx^4} + m\ddot{w} = 0 \tag{3}$$

Placing the assumed mode of Eq.(2) in the governing equation, Eq.(3) yields:

$$EI \frac{d^4 \overline{w}}{dx^4} - m\omega^2 \overline{w} = 0 \tag{4}$$

The solution to this homogeneous equation is :

$$\overline{w}(x) = C_1 \sinh \lambda x + C_2 \cosh \lambda x + C_3 \sin \lambda x + C_4 \cos \lambda x \tag{5}$$

Where:

$$\lambda = \left(\frac{m\omega^2}{EI}\right)^{1/4}$$

Applying the boundary conditions for the simply supported beam, the solution can be found to be :

$$(6)\ \omega(x,t) = \sin\frac{\pi r x}{l} \sin\left(r^2 \pi^2 \sqrt{\frac{EI}{mL^4}} t\right)$$

Now, the attachments can be treated by Rayleigh method to give the upper bound to the fundamental frequency of:

$$(7)\ \omega_1^2 = \frac{g\sum_i m_i w_i}{\sum_i m_i w_i^2}$$

The lower bound to the fundamental frequency using Dunkerly equation is then made by:

$$\frac{1}{\omega_1^2} < \sum_i a_{ii} m_i \tag{8}$$

The value, number and distribution of substructures was assumed to happen according to the normal distribution criteria. For instance, when attaching one mass the mean of the natural frequencies was calculated by:

$$\text{Mean }(N) = (1 / n) \times \sum_{i=1}^{n} f_i \tag{9}$$

, the standard deviation :

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} \{f_i - f(N)\}^2} \tag{10}$$

and the normal distribution:

$$Z = \{ f - f(N) \} / \sigma \tag{11}$$

Where f is random variable

2.2.Modeling:

The master structure is modeled as BEAM3 which is a unit-axial element with tension, compression, torsion, and bending capabilities .The element has three degrees of freedom at each node translations in the x and y direction and rotation about the nodal z-axis.

For 1-DOF mass attachment model, the substructure is modeled as MASS21 which is a point element having up to six degrees of freedom : translations in the nodal x , y , and z directions about the nodal x , y and z axes . A different mass and rotary inertia may be assigned to each coordinate direction. .

For 1-DOF mass-spring attachment model, the substructure is modeled as COMBIN40 which is a combination of a spring –slider and damper in parallel , coupled to a gap in series . A mass can be associated with one or both nodal points.

The simply supported beam is considered as a main structure , and two types of attachment models have been tested, namely (1- DOF mass attachment) and (1-DOF mass – spring attachment) ; the number of attachment is to be between one to five and their location is considered to be (x = L/6 , L/3 , L/2 , 2L/3 , 5L /6) . **Fig.(3)** shows such a beam .This simply supported beam is modeled to seven nodes . The material properties and dimensions of the tested main structure are listed in **Table(1)**.Testing programs had been designed as shown in **Tables(2&3)** .

3.Results and Discussion:

The calculated changes in natural frequencies due to attaching one mass (m_0) at the positions, defined in **Table(2)**, were plotted in **Figs.(4 through 8)** for values of the added mass not exceeding (11%) of the original mass of the beam to avoid initial deformation or bending of the beam. In these figures, it can be noted that all natural frequencies decrease with increasing the added mass. The maximum change is equal to (10 %) for the first mode shape and with an attachment mass equal to (0.1073) of the beam mass (master structure mass).

Some of the natural frequencies are unchanged due to the change in the attachment mass and location , like the sixth natural frequency which was unchanged in all cases where the attachment is at or near the node points (zero deflection) of these mode shapes. .

The calculated change in natural frequencies due to attaching one (mass-spring) at the positions defined in **Table(3)** , were plotted in **Figs.(9 through 13)** ; values of the mass (m_0)

and stiffness (k_0) of the spring are chosen equal to the first mode of the beam (without any attachment which is equal to 28.759 Hz) .Again the added mass does not exceed (11%) of the original mass of the beam . In these figures, it is shown that the 1-DOF mass-spring attachment model has an effect on the first natural frequency only. The maximum change in natural frequencies is (20 %) for one mass-spring attachment model with an attachment mass equal to (0.1073) of the beam mass and spring stiffness equal to (0.7855) of the beam stiffness. Other natural frequencies are unchanged due to the change in the attachment mass-spring model .

By comparing results for the two attachment models as shown in **Figs.(14 through 18)** , it can be noted that 1-DOF mass-spring model has a greater effect on the first natural frequency by nearly 10% than the effect of 1-DOF mass model .

Engineering Statistics and normal distribution have been used to find the best distribution of the attachments. Thus, the natural frequencies for unknown number of attachments are shown in **Fig.(19)** and the probability of distribution of these attachments is plotted in **Fig.(20).** To find the best distribution of four masses with probability (40 %): From Fig.(20) , f1/F1 = 0.9657 and the value of the attaching masses are ($4m_1/ M = 0.0719$) from **Fig.(7)**. And for all cases one can follow this procedure to determine the value of attaching mass for known number of masses.

To determine the number of attaching masses with probability (40%): From **Fig.(20),** $f_1/F_1=0.9584$; From **Fig.(19)** the ratio(m_0/M) lies between (0.06869 – 0.07298) and from **Fig.(18)** the best number of masses are (5).

The natural frequencies for unknown number of mass-spring attachments for the first natural frequencies are plotted in **Fig.(21),** and the probability of distribution these attachments is plotted in **Fig.(22).** To find the best distribution of four (masses–springs)with probability (40%): From **Fig.(22),** the value of $f_1/F_1 = 0.8779$, and the values of the attaching masses are ($4m_1/ M = 0.06869$) from **Fig.(12)** and the value of the attaching spring stiffness ($4k_1 = 91420$ N/m). And for all cases one can follow this procedure to determine the value of attaching (mass-spring) for known number of masses.

To determine the number of attaching masses and springs with probability (40 %) : From **Fig.(22)** $f_1/F_1=0.8663$ and from **Fig.(21)** the mass (m_0/M) lies between (0.0644 - 0.06869) and from **Fig.(18)** the best number of masses is (5).

4.Conclusions:

The main conclusions that might be drawn are:

1- The additional substructure is a very important parameter in dynamic analysis of structures.

2- All natural frequencies decrease with increasing the added mass.

3- The effect of attachments when modeled as (mass-spring substructure) is greater than the effect of attachments when modeled as (mass substructure) for the same attachment mass.

4- The proposed statistical approach can be used to find the best distribution of attachments according to the probability of the normal distribution so that one can find the best number, value and location of the additional substructure .

5.References:

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6.Nomenclature:

A	area of the beam (m^2)
As	shear area (m^2)
E	Young´s modulus (N/ m^2)
f1 ,f2,	natural frequencies of the beam with attachment
P	force (N)
F_1,F_2,	natural frequencies of the beam without any attachment (rad / sec)
G	shear modulus (N/ m^2)
H	width (m)
I	area moment of inertia (m4)
i , j	integer
k	spring constant (N / m)
L	length (m)
M	mass of beam (kg)
m_1, m_2,	attaching mass (kg)
{Me}	The element matrix
M(x)	mass per unit length (kg / m)
N	mean
w	transverse deflections (m)
ω_n	the natural frequency of vibration(rad/sec)
x, y , z	Cartesian coordinates, displacement
ρ	mass density (kg / m^3)
{ }	matrix
r	radius of gyration(m)

Table (1): The Material properties and dimensions of the main structure

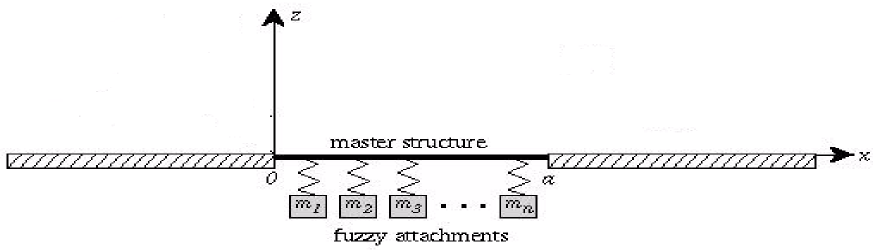
	Value	Unit
E	206.844E9	N/m²
A	2.58064E-3	m²
H	5.08E-2	m
L	2.032	m
ρ	7778	Kg/m³
I	5.55E-7	m^4
K	656759	N/m

Table (2): Testing program for mass attachment model

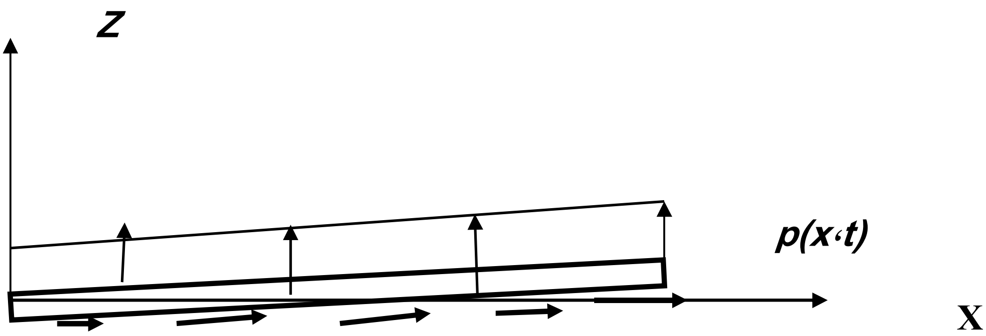
No. of additional masses	Value of additional mass(kg)	Value of one additional mass / beam mass m_0 / M		Position of additional masses
		Start value	final value	
1	$m_0 = 0.175$	0.02146	0.1073	L / 2
2	$m_1 = m_2 = 0.0875$	0.01073	0.05365	L / 3 , 2L / 3
3	$m_1 = m_2 = m_3 = 0.05833$	0.0071533	0.035766	L / 3 , L / 2 , 2L / 3
4	$m_1 = m_2 = m_3 = m_4 = 0.04375$	0.005365	0.026825	L / 6 , L / 3 , 2L / 3 , 5L / 6
5	$m_1 = m_2 = m_3 = m_4 = m_5 = 0.035$	0.004292	0.02146	L / 6 , L / 3 , L / 2 , 2L / 3 , 5L / 6

Table (3): Testing program for mass-spring attachment model

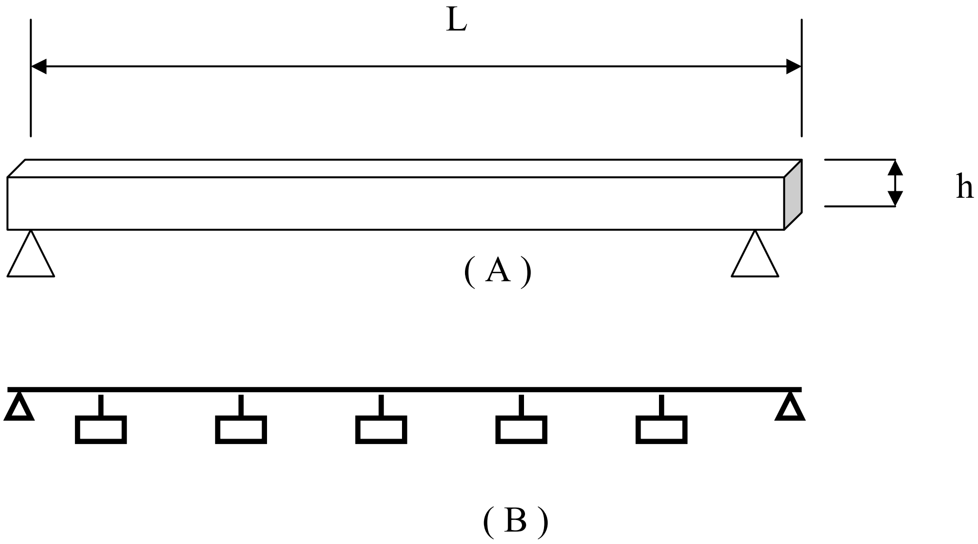
No. of additional masses-spring	Value of additional spring stiffness	Value of additional mass (kg)	Value of one additional mass / beam mass		Position of additional masses
			Start value	final value	
1	Kop	$m_0 = 0.175$	0.02146	0.1073	L / 2
2	$k_1 = k_2$	$m_1 = m_2 = 0.0875$	0.01073	0.05365	L / 3 , 2L / 3
3	$k_1 = k_2 = k_3$	$m_1 = m_2 = m_3 = 0.05833$	0.0071533	0.035766	L / 3 , L / 2 , 2L / 3
4	$k_1 = k_2 = k_3 = k_4$	$m_1 = m_2 = m_3 = m_4 = 0.04375$	0.005365	0.026825	L / 6 , L / 3 , 2L / 3 , 5L / 6
5	$k_1 = k_2 = k_3 = k_4 = k_5$	$m_1 = m_2 = m_3 = m_4 = m_5 = 0.035$	0.004292	0.02146	L /6 , L / 3 , L / 2 , 2L / 3 , 5L / 6



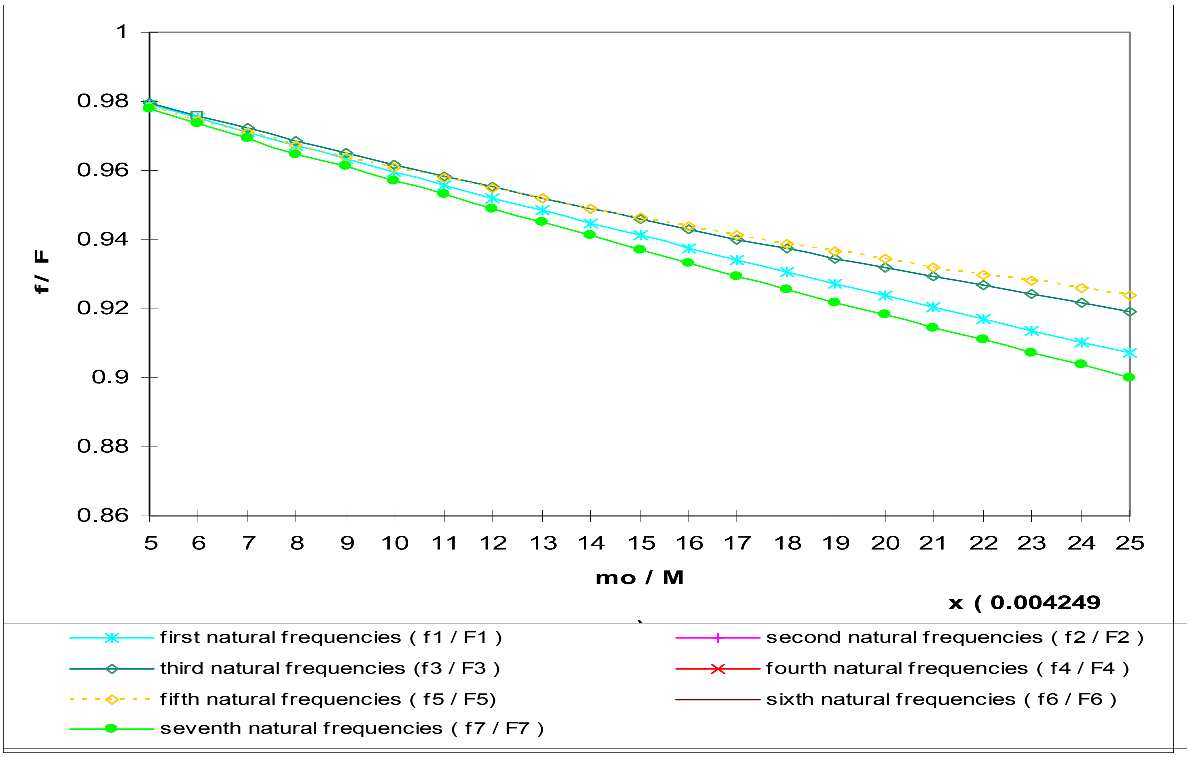
Figure(1): A simply supported beam with attachment



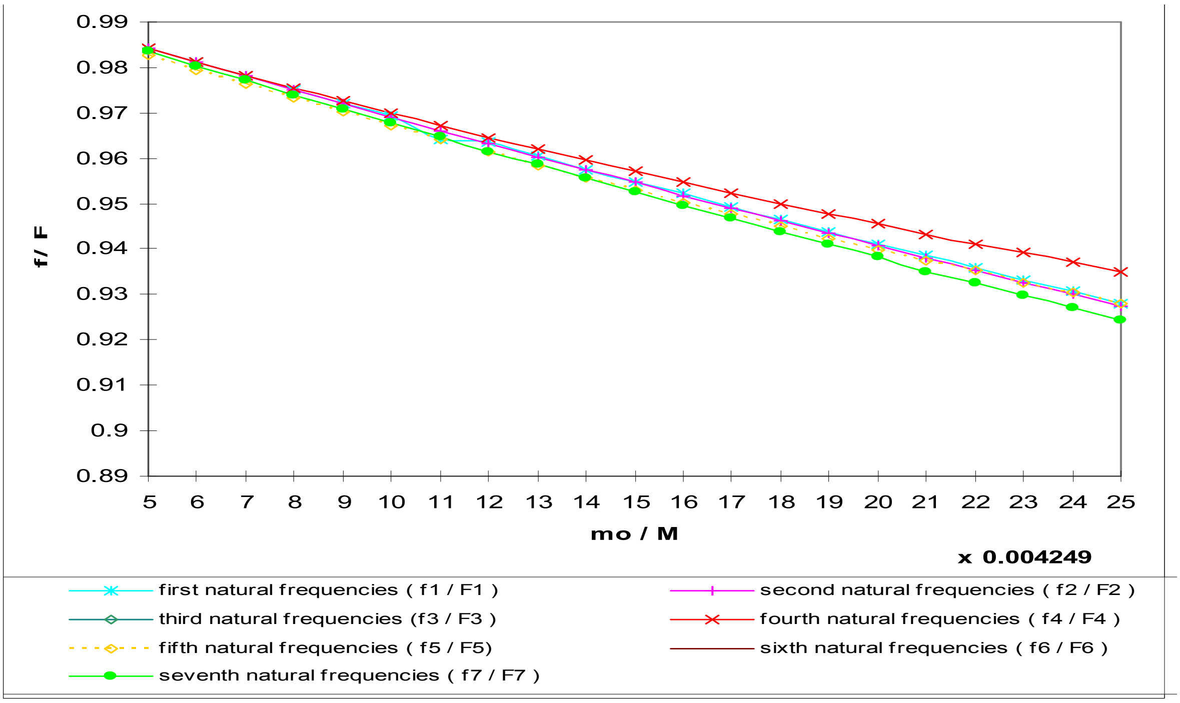
Figure(2): Representation of generalized beam-column



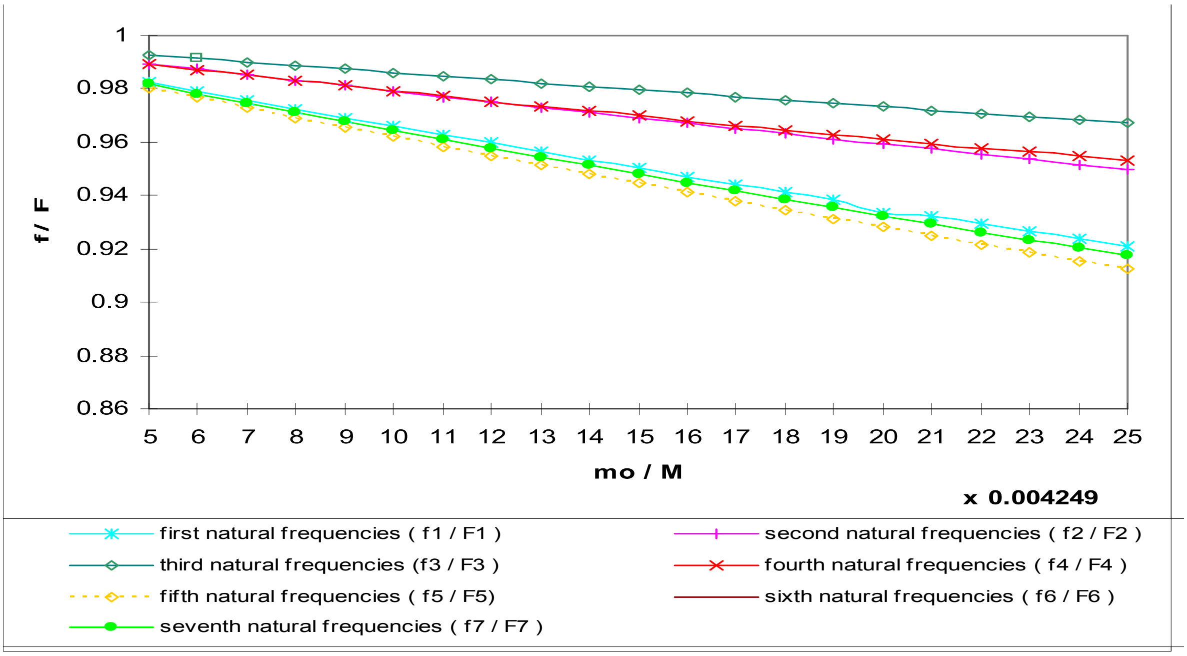
Figure(3): A-simply supported beam without attachment
B- simply supported beam with attachment



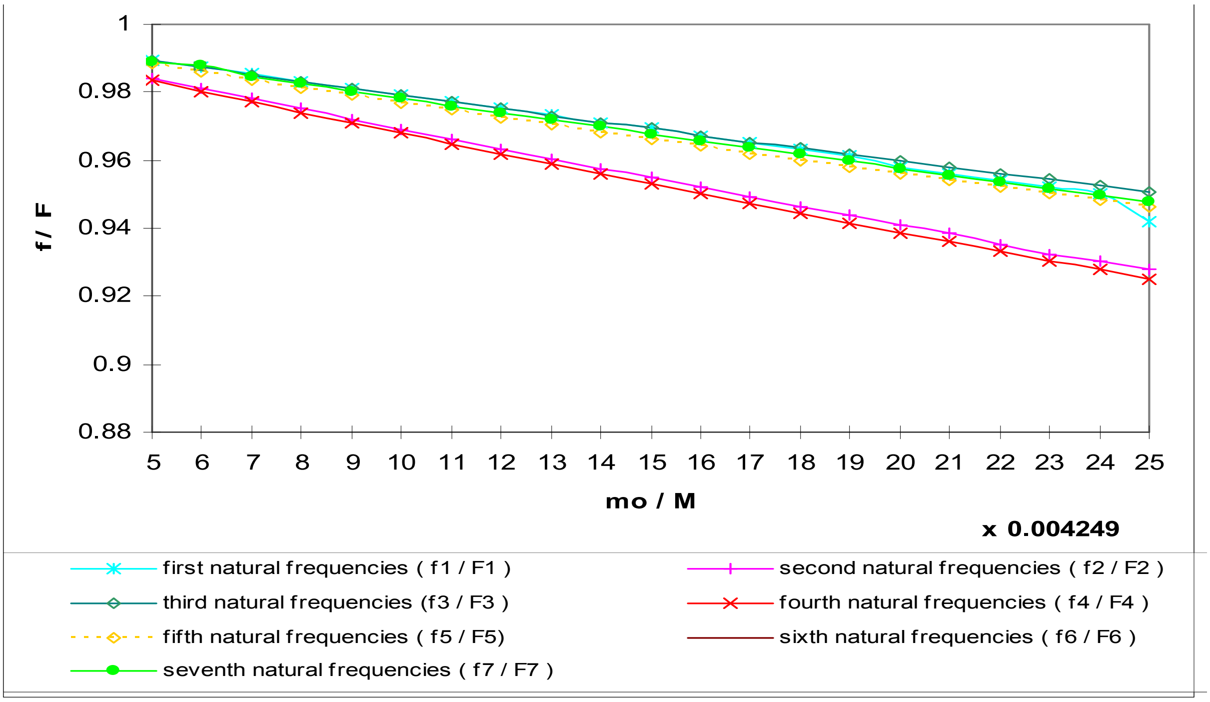
Figure(4): Effect of one additional mass at position x=L/2 on the natural frequency



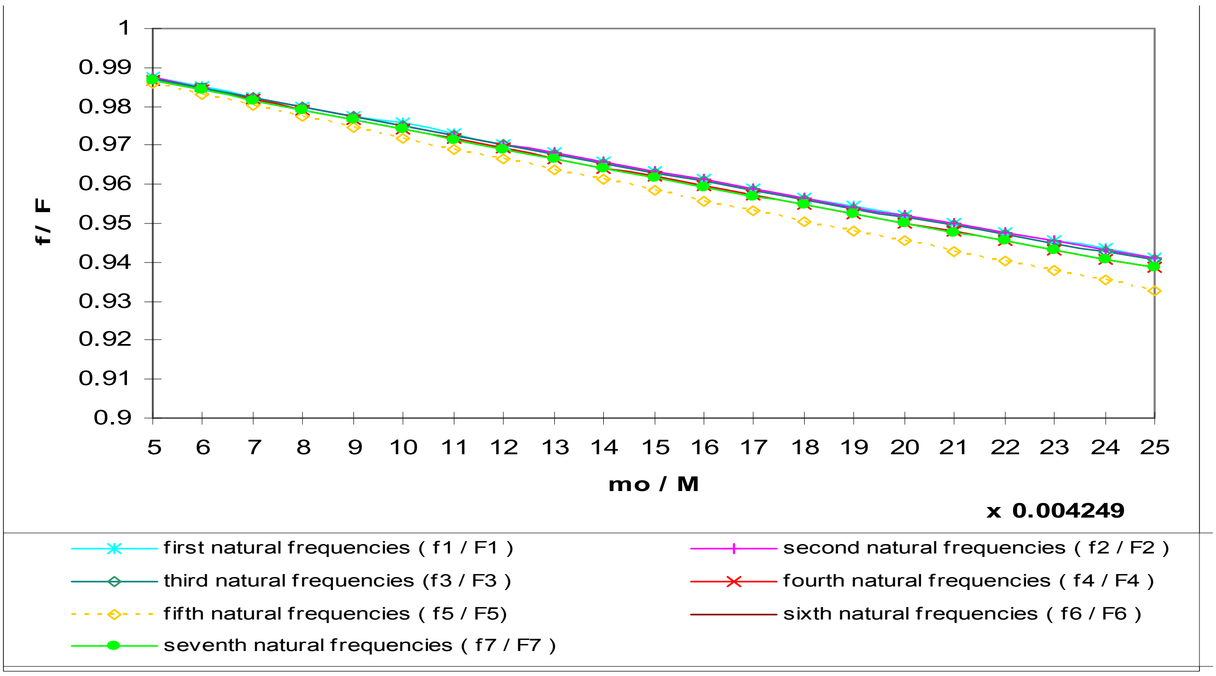
Figure(5): Effect of two additional masses at position x=L/3 , x=2L/3 on the natural frequency



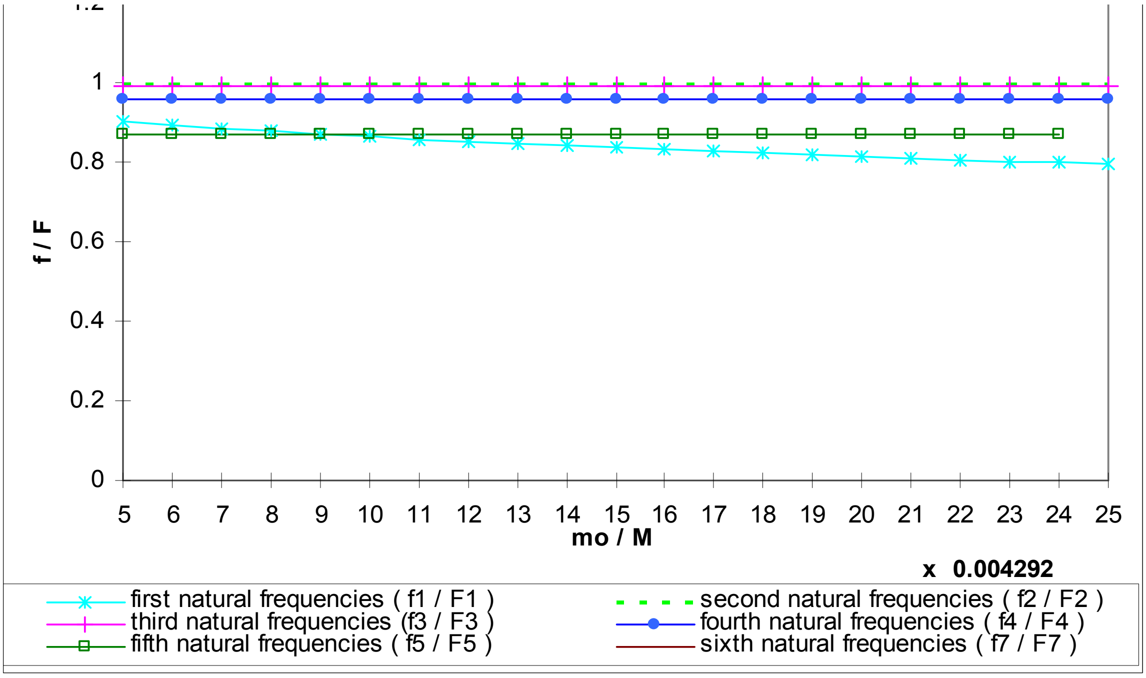
Figure(6): Effect of three additional masses at position x=L/3 on the natural frequency



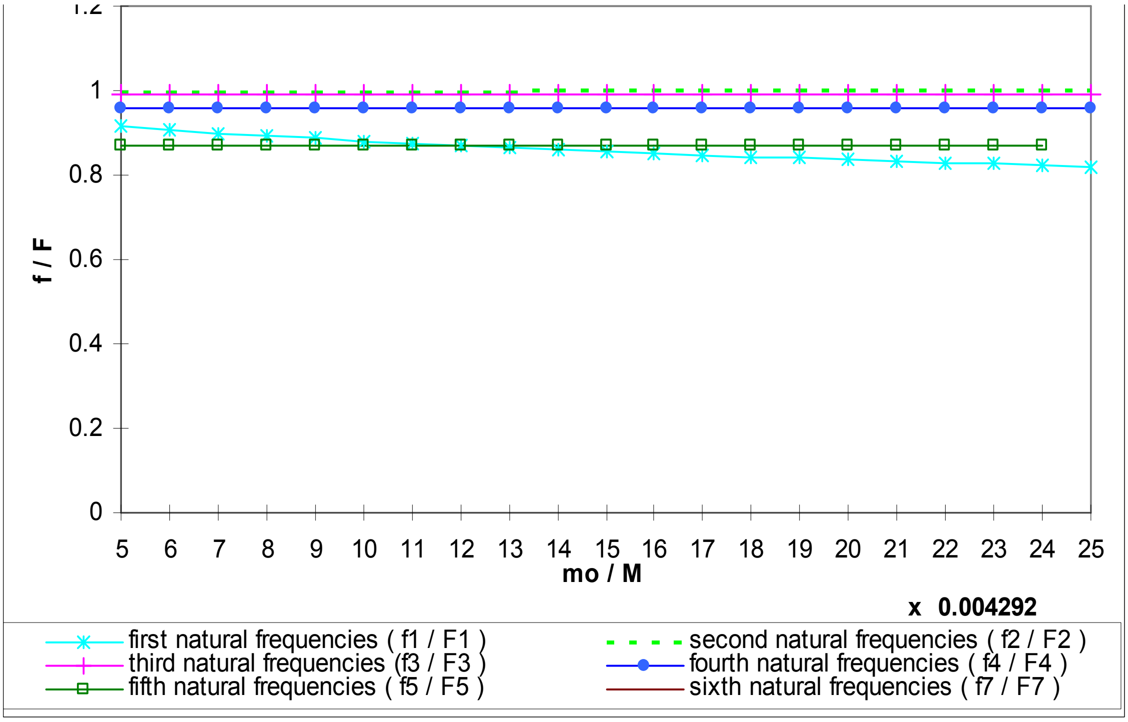
Figure(7): Effect of four additional masses at positions x=L/6,L/3,5L/6,2L/3, on the natural frequency



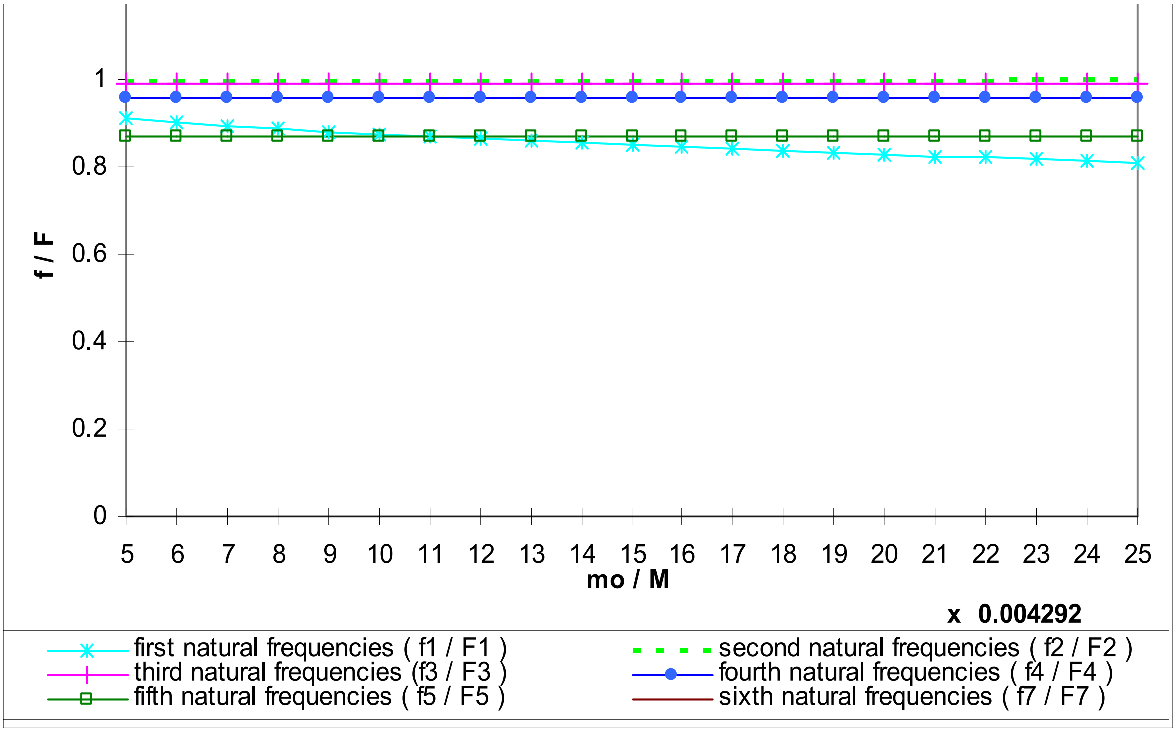
Figure(8): Effect of five additional masses at positions x=L/6,L/3,L/2,5L/6,2L/3, on the natural frequency



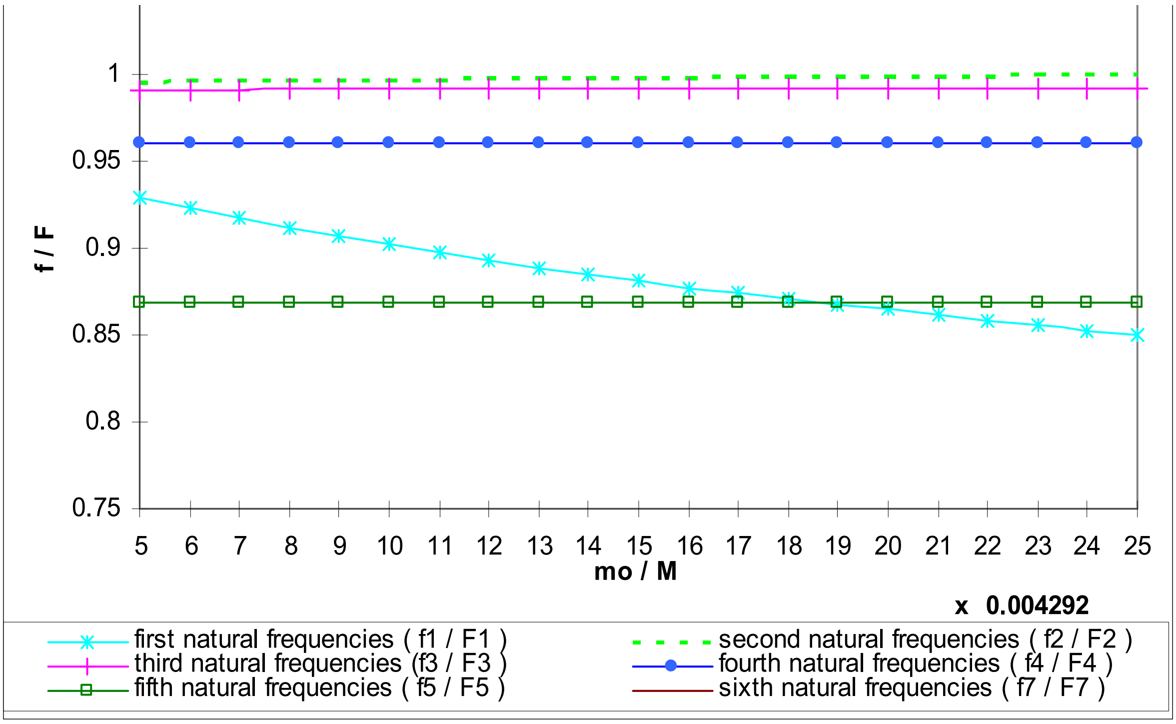
Figure(9): Effect of one additional mass-spring at position x=L/2 on the natural frequency



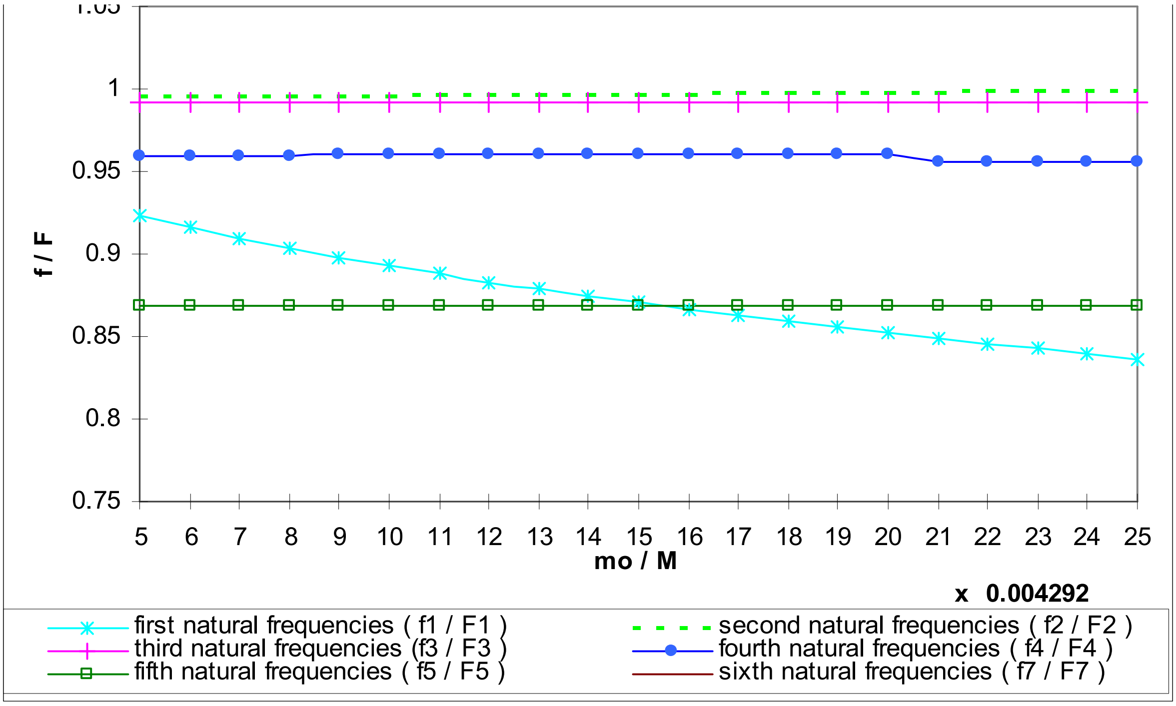
Figure(10): Effect of two additional mass-spring at position x=L/3,2L/3 on the natural frequency



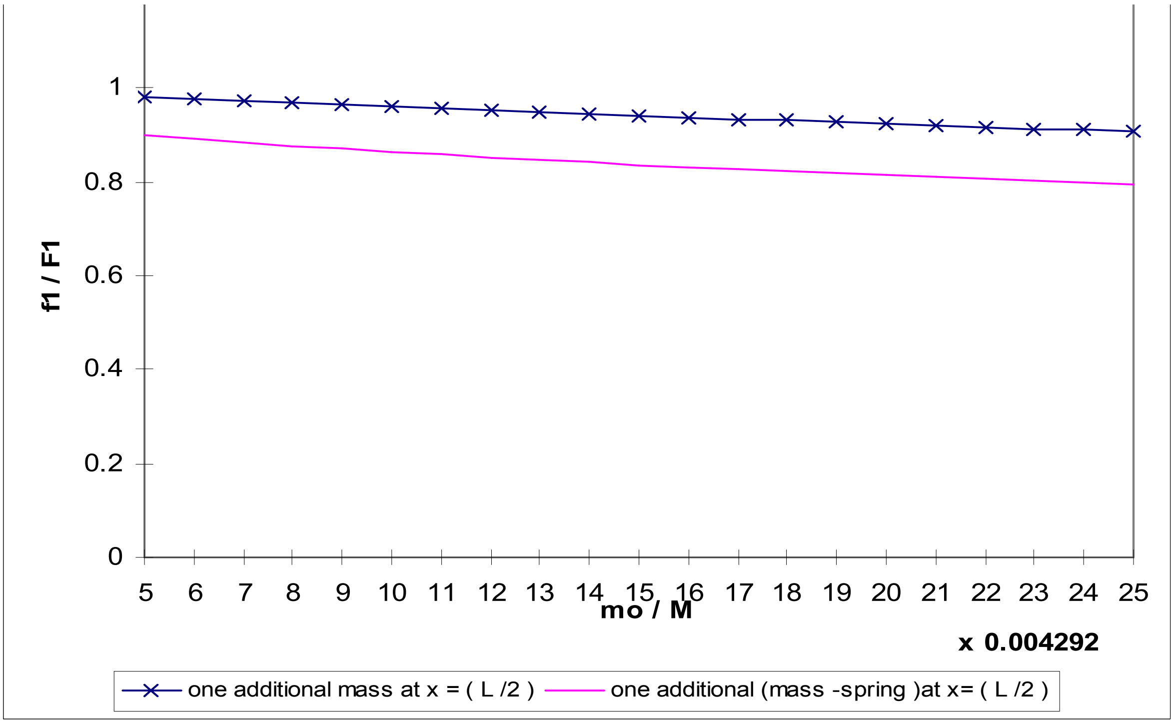
Figure(11): Effect of three additional mass-spring at positions x=L/3,L/2,2L/3 on the natural frequency



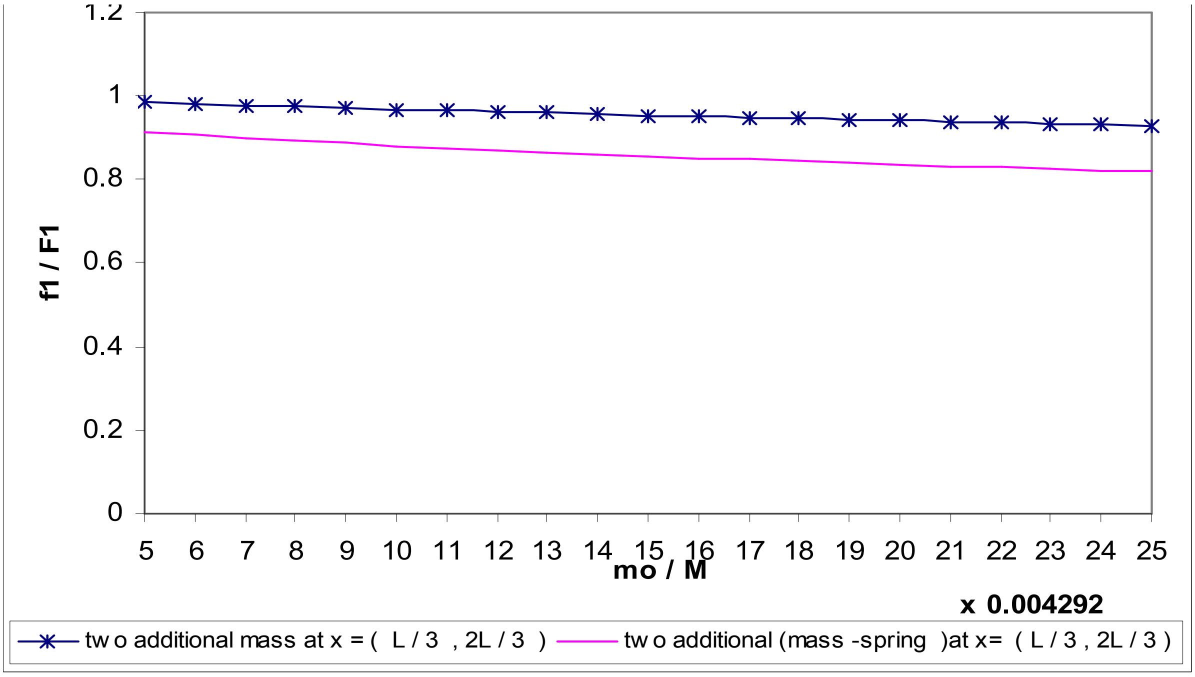
Figure(12): Effect of four additional mass-spring at positions x=L/6,L/3,2L/3,5L/6 on the natural frequency



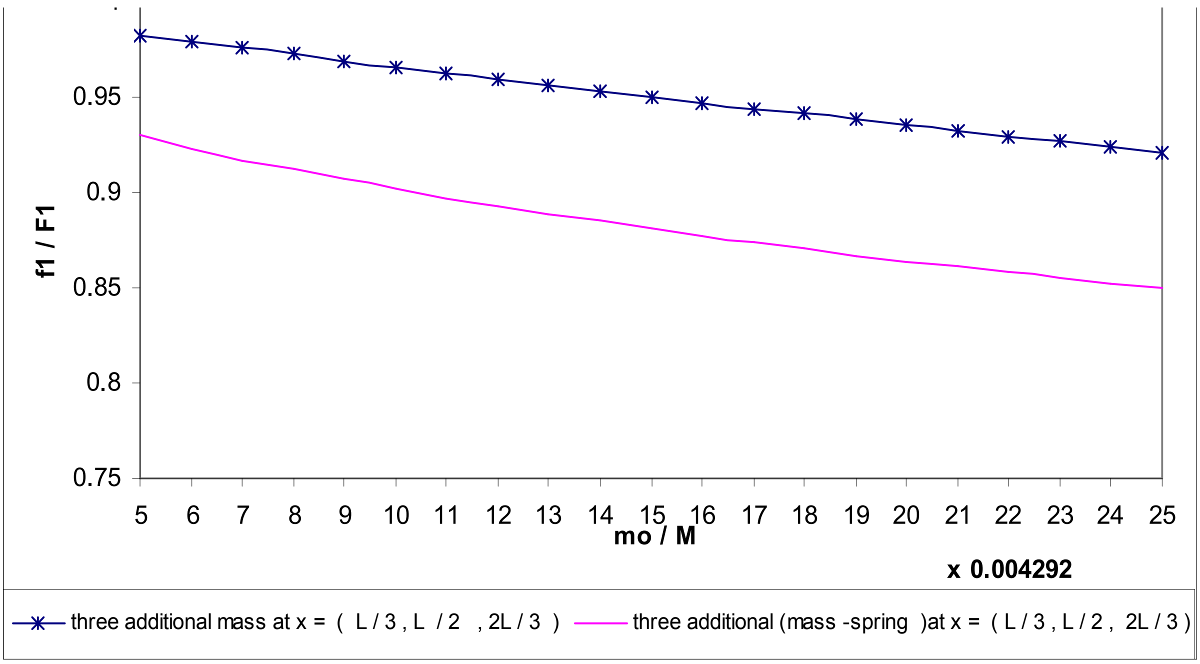
Figure(13): Effect of five additional mass-spring at positions x=L/6,L/3,L/2,2L/3,5L/6 on the natural frequency



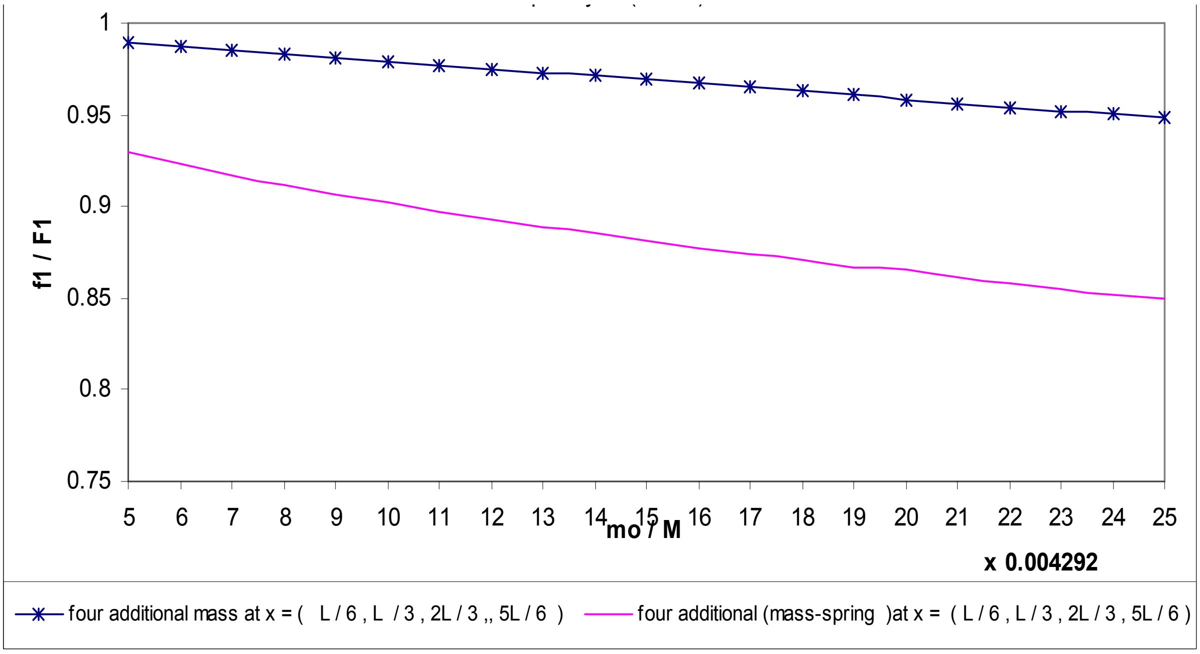
Figure(14): Effect of one additional mass, mass-spring model on the first natural frequency



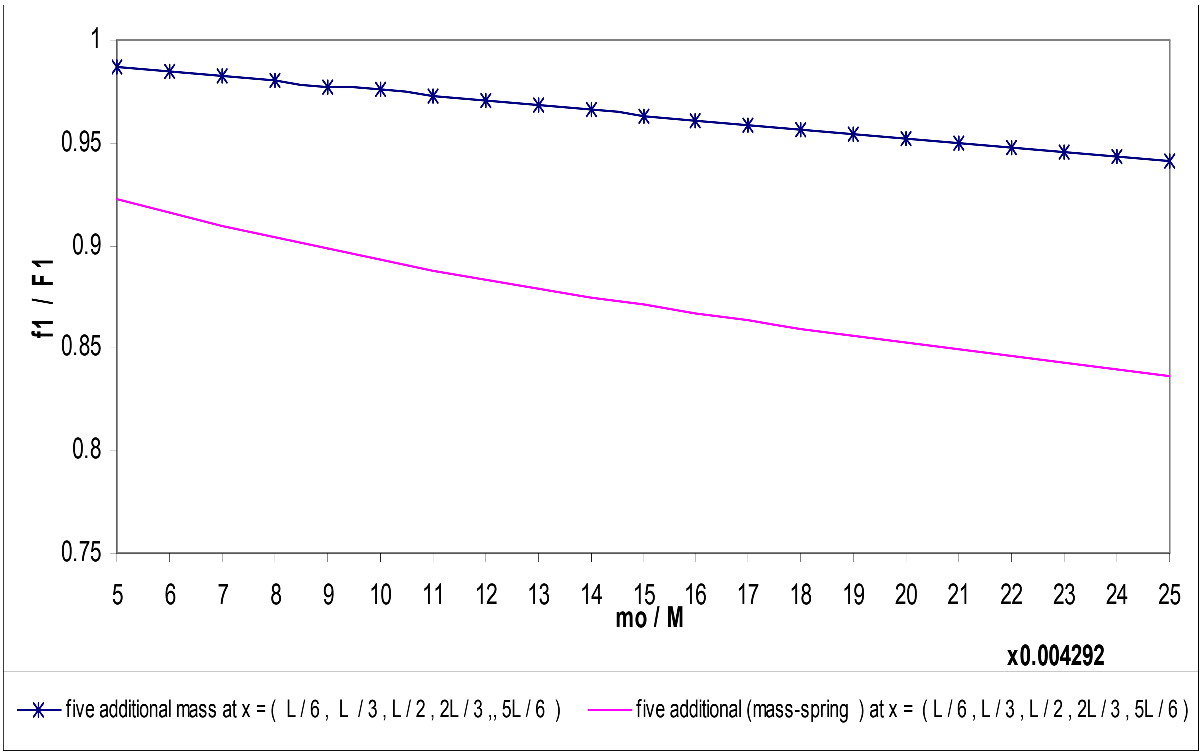
Figure(15): Effect of two additional mass, mass-spring model on the first natural frequency



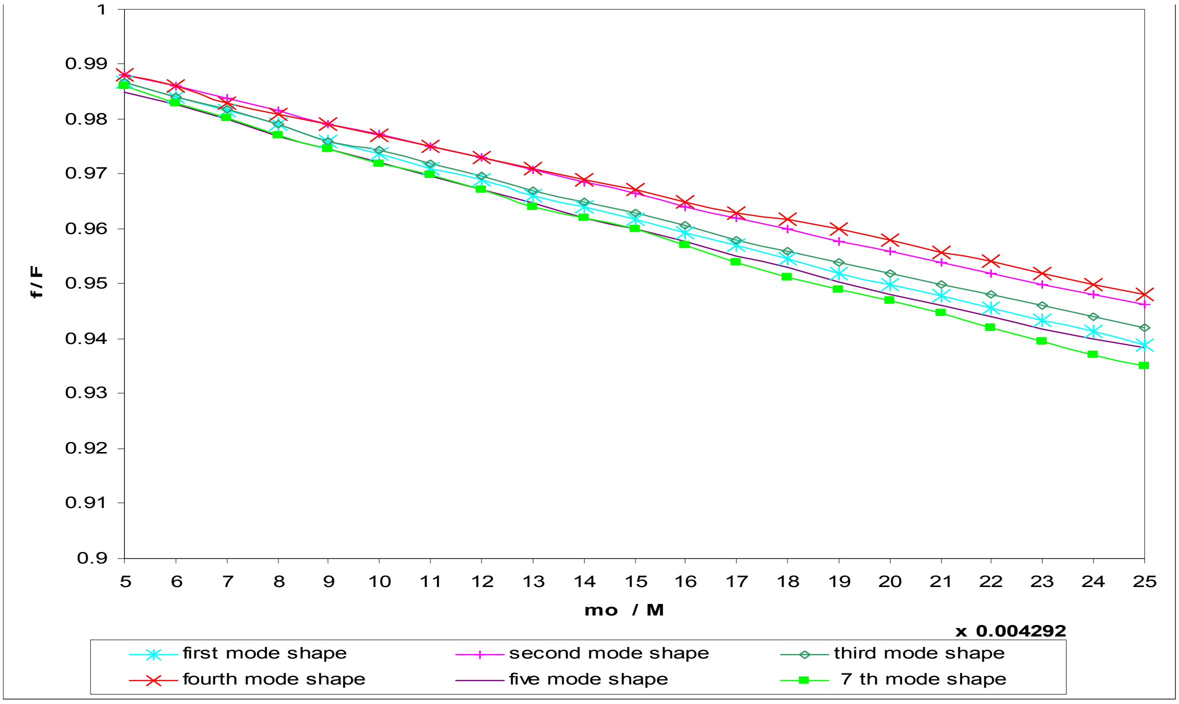
Figure(16): Effect of three additional mass, mass-spring model on the first natural frequency



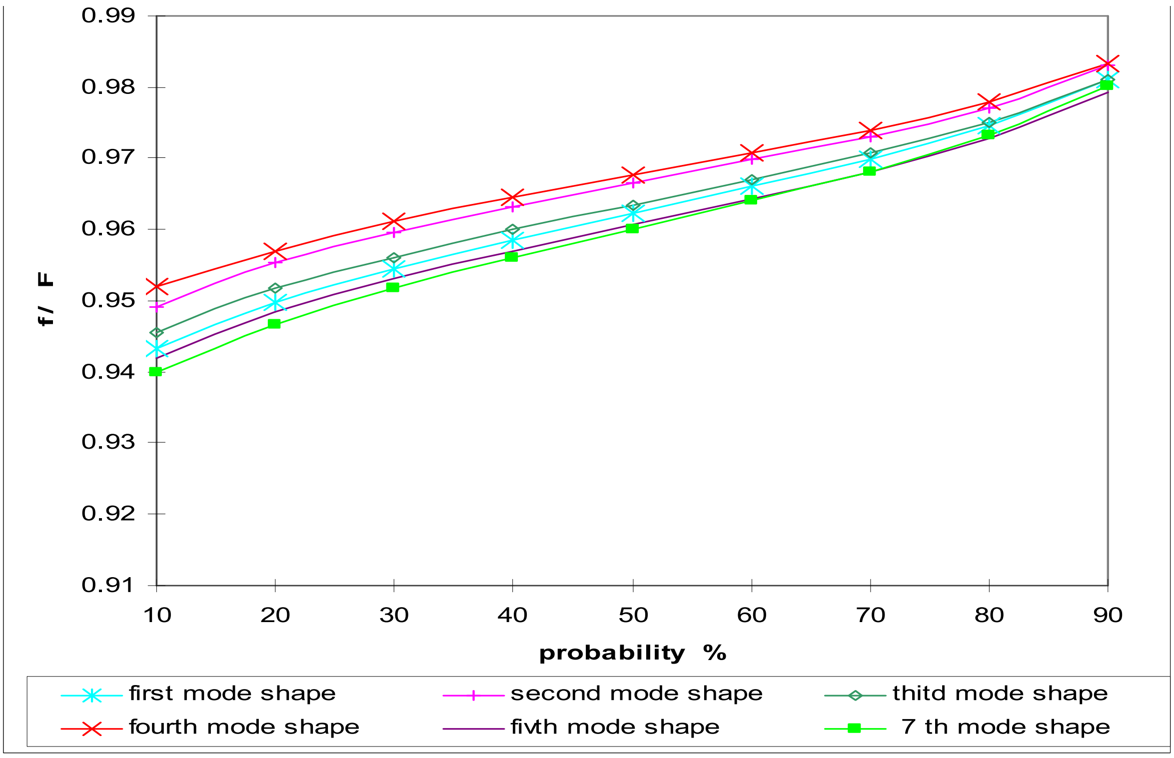
Figure(17): Effect of four additional mass, mass-spring model on the first natural frequency



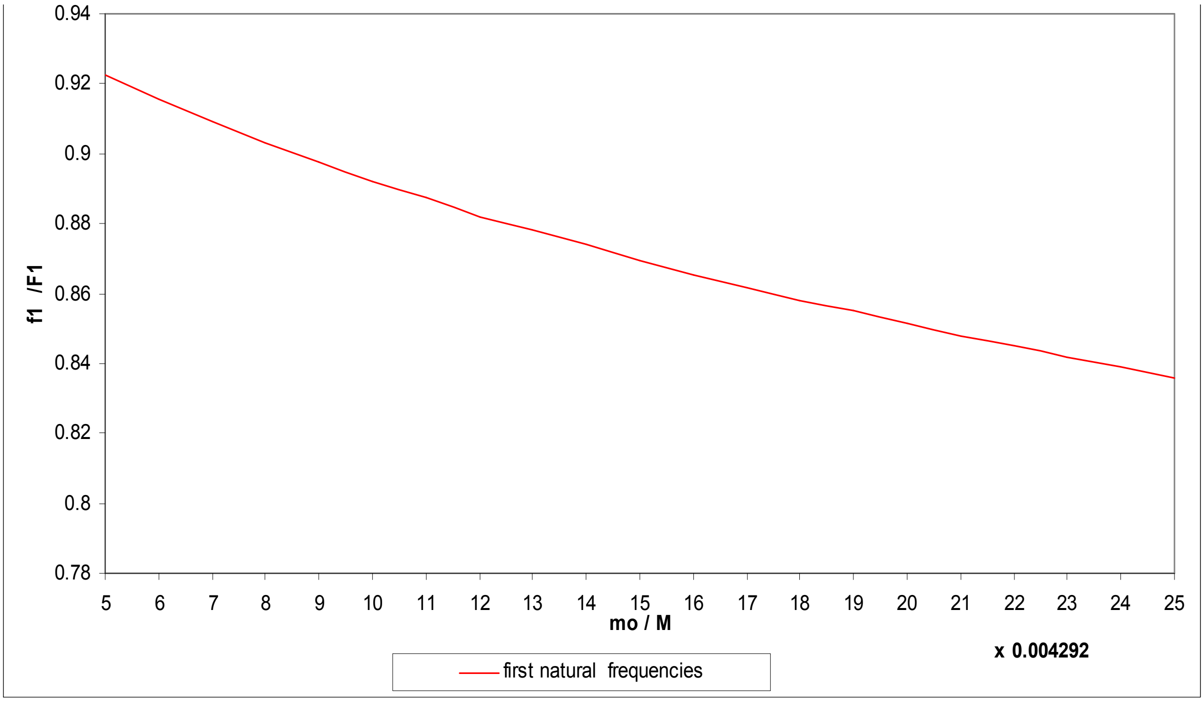
Figure(18): Effect of five additional mass, mass-spring model on the first natural frequency



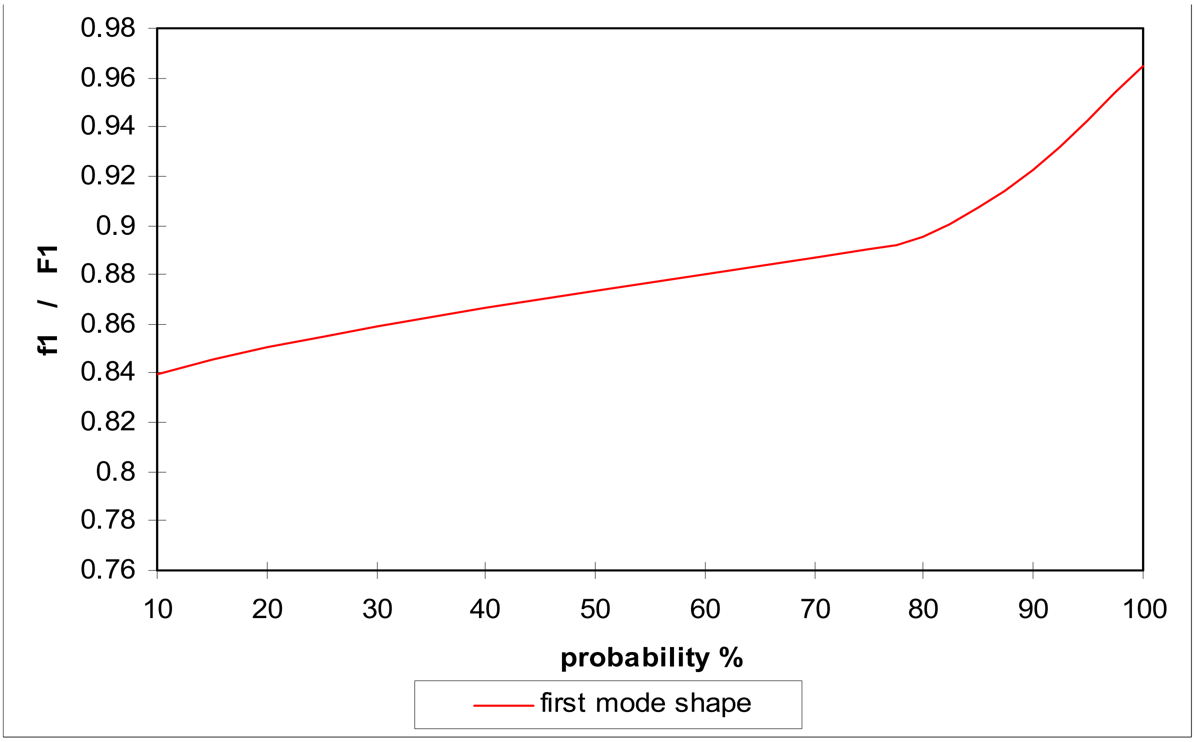
Figure(19): Effect of the number of additional masses on the natural frequency



Figure(20): Probability of the number of additional masses



Figure(21): Effect of the number of additional mass-springs on the first natural frequency



Figure(22): Probability of the number of additional mass-springs

تحليل الأهتزاز الحر لنظام متعدد الأجسام

د.بهاء ابراهيم كاظم　　　د.رياح نجم كطر　　　حسام محمد دولة
جامعة بغداد/كلية الهندسة　جامعةالأنبار/كليةالهندسة

الخلاصة:
استخدمت في هذا البحث عتبة بسيطة الأسناد كهيكل رئيسي بعدد غير محدد من الهياكل الثانوية (الملحقات) التي تم تمثيلها كنظام ذو درجة حرية واحدة. تم دراسة نموذجين من الملحقات نموذج يتألف من كتلة ملحقة ذات درجة حرية واحدة و نموذج يتألف من كتلة ملحقة ونابض ذاو درجة حرية واحدة.
وجد ان نوع الهياكل االثانوية مهم في في التحليل الديناميكي للهيكل الرئيسي وأن تاثير النموذج الملحق على الترددات الطبيعية للهيكل الرئيسي عندما يكون نموذج يتألف من كتلة ملحقة ونابض ذاو درجة حرية واحدة اكبر من تاثيره عندما يكون من كتلة ملحقة ذات درجة حرية واحدة لنفس الكتلة الملحقة.
تم أستخدام الاحصاء الهندسي و التوزيع الطبيعي لايجاد قيمة الملحقات التي يجب اضافتها الى العتبة لتحسين الخواص الحركية للهيكل الرئيسي و ايجاد افضل توزيع لهذه الملحقات .أظهرت النتائج بان توزيع الهياكل المضافة تسبب تغيرا" كبيرا" في الترددات الطبيعية بحيث يمكن لهذا الاسلوب الأحصائي المقترح ان يستخدم لايجاد افضل توزيع للملحقات وتحديد عدد وقيمة وموقع الملحقات.