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BEHAVIOR OF MULTI-LAYER COMPOSITE CONTINUOUS BEAMS WITH PARTIAL INTERACTION

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ABSTRACT

In this study an attempt is made to develop a method of analysis dealing with a multi-layer composite continuous beam , for linear material and shear connector behavior in which the slip (horizontal displacement) and uplift force (vertical displacement) are taken into consideration. The cross-sectional area for the beam consists of three layers varying in thickness and shear stiffness. The analysis is based on a approach presented by Roberts[1], basically for two layer simply supported beam, under uniform and point loads , which takes into consideration horizontal and vertical displacement in interfaces. The analysis led to a set of eight differential equations containing derivatives of the fourth and third order. A program based on the present analysis is built using finite difference method using boundary conditions. A comparison between the present analytical solution and previous studies shows close agreement. Continuous composite beams are very important element in construction of high rise buildings , multi-story frames and bridges, due to great advantages that can be obtained by using this sort of structural elements, such as reducing the beam moments, suitable reduction in deflections. The model deals with continuous beam consisting from three layers as a cross-sectional area with inter-layer slip. The cross-sectional area consist of composite material including intermediate layer from concrete and an upper and lower material with high strength in tension and compression (i.e. steel plates or steel beams)

KEYWORDS: Composite , Multi-layer , Continuous , Partial , Interaction

1. INTRODUCTION

Composite construction has been widely used for building construction over the past 60 years, developed initially for most structural elements due to the advantages provided by such types of elements. A perfect connection between the components of composite elements (mostly steel, concrete and timber) exists only theoretically. Experimental investigation has shown that significant slip occurs at the interface between these components, even when a large number of connectors are proved. Some types of connectors give a very rigid connection, others are more deformable in which a certain slip is inevitable. This problem is more complicated when fewer connectors than the number required for full interaction are used. The modification in the behavior of a composite beam by the presence of slip was illustrated by analysis conducted by many researchers. These analyses led to differential equations (number of these equations depending on the degree of freedom) that are to be solved fresh for each type of loading and the variation in dimensions or properties of beams. Multi-layer composite beam (also called laminated beam structures) are very important structures and relatively new and are used not in civil engineering only but in many industries such as aircraft and marine engineering. The first interaction theory that takes account of slip effects was initially formulated by Newmark [2], based on elastic analysis of composite beams assuming linear material and shear connector behavior.

Adekola [3] formulated another equations , two equation , based on interaction theory, which takes account of slip, uplift and friction effect. Each component of a composite beam was assumed to behave separately in accordance with simple bending theory. In addition it was assumed that the rate of change of the axial force is directly proportional to slip, and uplift force is directly proportional to differential deflection. The equilibrium and compatibility relations lead to two differential equations of fourth order connecting the uplift tension arising from differential deflections of the two components of the composite beam with the axial force within each of the components. The equations contain derivatives of fourth order in uplift forces and second order in axial forces, and they were solved by finite difference method, in which they were rearranged such that unknowns exist at each node point of a simply supported composite beam. Obtaining the complete solution for the axial forces and uplift forces, deflections can then be determined.

Using the same element presented by Newmark, Johnson [4] in 1975 proposed a partial interaction theory for simply supported beams, in which the analysis was based on elastic theory. The composite beam was assumed to be linear elastic material. The discrete connection was assumed to be smeared along the beam, so that the connector strength and stiffness can be quoted per unit length of beam. In addition, the connector behavior was assumed linearly elastic. The effects of uplift were neglected, i.e. no gap between the two components of the composite beam exists and the same curvatures are used for them. Equations deduced from equilibrium, elasticity and compatibility were so arranged that a second order differential equation relating the slip at the interface to the distance along the beam were obtained. The solution of the equation gives the slip distribution along the beam, back substitution into the equilibrium and compatibility equations get the curvature distribution deflections and stresses along the beam. Both of the two approaches analyze two layers of composite beam with partial interaction and gives single, second order explicit differential equation. This equation must be solved for each type of loading to have the complete solution.

Roberts [1] presented an approach for the analysis of composite beam with partial interaction, in which the basic equilibrium and compatibility equations were formulated in terms of four independent variables, i.e. the axial displacements of the concrete and steel and the deflections of the two layers. Linear elastic materials and shear connector behavior were assumed with the concrete remaining uncracked, and both the slip and separation at the interface were incorporated. The analysis resulted in four differential equations, which contain derivatives of third order in axial displacements and fourth order in deflections. Numerical solutions of the basic equations were obtained by expressing them in finite difference form and the complete system of the equations, i.e. four per node, was solved for the unknown displacements and deflections. An application of the theory was made in which the behavior of a simply supported composite beam under service loading was studied. The normal stiffness of the shear connection per unit length was assumed infinite, i.e. no separation occurs and equal curvatures of the interaction components exist. The shear stiffness of the shear connections per unit length were varied such that uniform, triangular and discontinuous distribution of shear connectors were obtained. The basic equilibrium and compatibility equations were formulated in terms of four independent variables, i.e. the axial displacements and deflections of the layers, Linear elastic materials and shear connector behavior was assumed with the concrete remaining uncracked, and both the slip and separation at the interface were incorporated. The analysis resulted in four differential equations, which contain derivatives of third order in axial displacements and fourth order in deflections.

The previous approaches were devoted to two layer simply supported beam . In case of multi-layer continuous composite beam a certain modification must be taken in consideration.

2. THEORY

The analysis of composite continuous beam, consisting of three layers were considered. The cross-sectional area consisting of two layers of steel plates bounded a layer of reinforced concrete. There is a problem of negative moment occurs at the supports , so, the concrete layer is assumed crack, as shown in Fig. (1) .

2.1 Assumptions

The basic assumptions of conventional beam theory were used, where plane sections are assumed to remain plane. Also, the connection was assumed to have negligible thickness and possesses finite normal and tangential stiffness.

2.2 Equilibrium .

An element of a composite of three layers, length (dx), shown in Figure (1), is subjected to moments, (M), shear forces, (V), and axial forces, (F), subscripts a, b, and c denote, three layers from upper to lower layer, and the local x-z axes pass through the centroids of the materials. The beam is subjected to a uniform distributed load The equilibrium requirements led to the following equations:

$$dV_{sp1} + dV_r + \partial V_{sp2} = r d_x \qquad (1)$$

Dividing Eq. (1) by (d_x) and taking limits as (d_x) tends to zero, gives:

$$V_{sp1,x} + V_{r,x} + V_{sp2,x} = r \qquad (2)$$

Where subscripts x denotes differentiation. For live load and dead load, ρ is equal to :

$$r = r_{sp1} + r_c + r_{sp2} \qquad (3)$$

where r_{sp1}, r_{sp2} and r_c are the distributed self of the upper steel plate, lower steel plates and concrete layer respectively.

Taking moment about the origin of coordinates in the first layer , gives :

$$\begin{aligned} dM_{sp1} + dM_r + dM_{sp2} &= (V_{sp1} + V_r + V_{sp2})d_x + \\ &(dV_{sp1} + dV_r + dV_{sp2})\frac{d_x}{2} + dF_r.h_1 + dF_{sp2}(h_1 + h_2) \end{aligned} \qquad (4)$$

Where h_1 is the distance between the centroids of the steel plate (upper layer) and steel bars , h_2 is the distance between the centroids of the lower steel plate and steel bars. After neglecting the second order terms and dividing by d_x, Eq.(4) becomes :

$$M_{sp1,x} + M_{r,x} + M_{sp2,x} = V_{sp1} + V_r + V_{sp2} + F_{r,x}.h_1 + F_{sp2,x}(h_1 + h_2) \qquad (5)$$

Differentiating Eq. (5) gives:

$$M_{sp1,xx} + M_{r,xx} + M_{sp2,xx} = V_{sp1,x} + V_{r,x} + V_{sp2,x} + F_{r,xx}.h_1 + F_{sp2,xx}(h_1 + h_2) \tag{6}$$

and substituting Eq. (1) into Eq. (2) gives:

$$M_{sp1,xx} + M_{r,xx} + M_{sp2,xx} - F_{r,xx}.h_1 - F_{sp2,xx}(h_1 + h_2) = r \tag{7}$$

$$F_{sp1,x} + F_{r,x} + F_{sp2,x} = 0 \tag{8}$$

Taking moment about the origin of coordinate in the second layer, gives :

$$\begin{aligned} &dM_{sp1} + dM_r + dM_{sp2} = (V_{sp1} + V_r + V_{sp2})d_x + \\ &(dV_{sp1} + dV_r + dV_{sp2})\frac{d_x}{2} - dF_{sp1}.h_1 + dF_{sp2}.h_2 \end{aligned} \tag{9}$$

After neglecting the second order terms and dividing by d_x, Eq. (9) becomes :

$$M_{sp1,x} + M_{r,x} + M_{sp2,x} = V_{sp1} + V_r + V_{sp2} - F_{sp2}.h_1 + F_{sp2,x}.h_2 \tag{10}$$

Differentiating Eq. (10) gives:

$$M_{sp1,xx} + M_{r,xx} + M_{sp2,xx} = V_{sp1,x} + V_{r,x} + V_{sp2,x} - F_{sp2,x}.h_1 + F_{sp2,x}.h_2 \tag{11}$$

and substituting Eq. (1) into Eq. (11) gives:

$$M_{sp1,xx} + M_{r,xx} + M_{sp2,xx} + F_{sp1,xx}.h_1 - F_{sp2,xx}.h_2 = r \tag{12}$$

Eqs.(7),(8)and(12)are the three basic equilibrium equations required for the solution.

2.3 Compatibility

Assuming plane sections within each material remain plane, the total displacement of each layer at the interface denoted by U_{ati}, is given by :

$$U_{sp1ti} = u_{sp1} - z_{sp1i}.w_{sp1,x} \tag{13}$$

in which z_{sp1i} is the z-coordinate of the interface relative to the local x-z axes , u_{sp1} and w_{sp1} are the displacement of the concrete in the x and z direction.
Similarly for the steel bars and lower steel plate.

$$U_{rti} = u_r - z_{ri}.w_{r,x} \tag{14}$$

$$U_{sp2ti} = u_{sp2} - z_{sp2i}.w_{sp2,x} \tag{15}$$

The slip , U_{sp1r}, at the interface between the upper two layers is denoted as the relative displacement in the x-direction of initially adjacent particles , hence :

$$U_{sp1r} = u_{sp1ti} - u_{rti} \quad (16)$$
$$U_{rsp2} = u_{rti} - u_{sp2ti} \quad (17)$$

combining Eqs. (13) , (14) , (15) (16) and (17) gives :

$$U_{sp1r} = (u_{sp1} - z_{sp1i}.w_{sp1,x}) - (u_r - z_{ri}.w_{r,x}) \quad (18)$$
$$U_{rsp2} = (u_r - z_r.w_{r,x}) - (u_{sp2} - z_{sp2i}.w_{sp2,x}) \quad (19)$$

if the shear stiffness of the joint per unit length is denoted by k_s, the shear force per unit length at the interfaces q_1 and q_2 is :

$$q_1 = k_{s1}.U_{sp1r} \quad (20)$$
$$q_2 = k_{s2}.U_{rsp2} \quad (21)$$

and considering the equilibrium of the upper layer in the x-direction gives:

$$F_{sp1,x} = q_1 \quad (22)$$
$$F_{r,x} = q_2 - q_1 \quad (23)$$
$$F_{sp2,x} = -q_2 \quad (24)$$

substituting for U_{sp1r} and U_{rsp2} from Eq. (18) and (19) givers:

$$F_{sp1,x} - k_{s1}[(u_{sp1} - z_{sp1i}.w_{sp1,x}) - (u_r - z_{ri}.wr, x)] = 0 \quad (25)$$

$$\begin{aligned} &F_{r,x} + k_{s1}[(u_{sp1} - z_{sp1i}.w_{sp1,x}) - (u_r - z_{ri}.wr, x)] \\ &- k_{s2}[(u_r - z_r.w_r) - (u_{sp2} - z_{sp2}.w_{sp2})] = 0 \end{aligned} \quad (26)$$

The separation at the interface between any two layers, w_{rsp1} and w_{rsp2} is the relative displacement in the z-direction of initially adjacent particles i.e:

$$w_{rsp1} = w_r - w_{sp1} \quad (27)$$
$$w_{sp2r} = w_{sp2} - w_r$$

If P_1 and P_2 denotes the normal force per unit length at the interfaces , equilibrium of the upper steel plate in the z-direction gives ; (as shown in Fig.(1-c)):

$$V_{sp1,x} = r_i + r_{sp1} + P_1 \tag{29}$$

In the second layer

$$V_{r,x} = r_r + r_c + P_2 - P_1 \tag{30}$$

$$V_{sp2,x} = r_{sp2} - P_2 \tag{31}$$

Considering the moment equilibrium of the upper layer about the origin of coordinates gives :

$$V_{sp1} = M_{sp1,x} + q_1.z_{sp1i} \tag{32}$$
$$V_r = M_{r,x} + q_1.z_{ri} + q_2.z_{ri} \tag{33}$$
$$V_{sp2} = M_{sp2,x} + q_2.z_{sp2i} \tag{34}$$

Hence, combining Eqs. (22), (24) , (29) ,(31) , (32) and (33) gives:

$$V_{sp1,x} = M_{sp1,xx} + q_{1,x}.z_{sp1i} \tag{35}$$

$$V_{r,x} = M_{r,xx} + q_{1,x}.z_{ri} + q_{2,x}.z_{ri} \tag{36}$$

$$V_{sp2,x} = M_{sp2,xx} + q_{2,x}.z_{sp2i} \tag{37}$$

$$r_i + r_{sp1} + P_1 = M_{sp1,xx} + F_{sp1,xx}.z_{sp2i} \tag{38}$$

$$P_1 = M_{sp1,xx} + F_{sp1,xx}.z_{sp2i} - (r_i + r_{sp1}) \tag{39}$$

$$P \ - P_1 = M_{r,xx} - F_{sp2,xx}.z_{ri} + F_{sp1,xx}.z_{ri} - r_c \tag{40}$$

$$r_{sp2} - P_2 = M_{sp2,xx} + q_{2,x}.z_{sp2i} \tag{41}$$

$$P_2 = -M_{sp2,xx} + F_{sp2,xx}.z_{sp2i} + r_{sp2} \tag{42}$$

if the normal stiffness of the joint per unit length is denoted by k_{n1} and k_{n2} , then :

$$P_1 = k_{n1}.w_{rsp1} = k_{n1}(w_r - w_{sp1}) \tag{43}$$

$$P_2 = k_{n2}.w_{sp2r} = k_{n2}(w_{sp2} - w_r) \tag{44}$$

substituting for P_1 and P_2 from Eq. (40) into Eq. (43) and (44) gives:

$$k_{n2}(w_{sp2}-w_r)-k_{n1}(w_r-w_{sp1})=M_{r,xx}-F_{sp2,xx}.z_{ri}+F_{sp1,xx}.z_{ri}-r_c \tag{45}$$

$$M_{r,xx}-F_{sp2,xx}.z_{ri}+F_{sp1,xx}.z_{ri}k_{n2}(w_{sp2}-w_r)-k_{n1}(w_r-w_{sp1})=r_c \tag{46}$$

Eqs. (25) , (26) and (46) are the three basic compatibility equations required for the complete solution.

3. BASIC DIFFERETIAL EQUATIONS

From the analytical model, the six independent differential equations (equilibrium and compatibility), may be expressed in terms of displacement variables, $(u_{sp1}, w_{sp1}, u_r, w_r, u_{sp2})$ and (w_{sp2}) as follows: Assuming plane sections within each material remain plane, the axial strain (e) can be expressed in terms of displacements $(u\ ,w\ )$ relative to the local x and z –axes, which are assumed to pass through the centroid of the three materials. Hence:

$$e_{sp1}=U_{sp1t,x}=U_{sp1,x}-z_{sp1}.w_{sp1,xx} \tag{47}$$
$$e_r=U_{rt,x}=U_{r,x}-z_r.w_{r,xx} \tag{48}$$
$$e_{sp2}=U_{sp2t,x}=U_{sp2,x}-z_{sp2}.w_{sp2,xx} \tag{49}$$

These subscripts sp1, r and sp2 denote the upper steel plate, steel bars and lower steel plats. Subscript (x), denotes differentiation and (z) the distance from the origin of coordinates to the limits of the layers.

Stresses now can be related to strain via the material properties (E_{sp1}, E_r) and (E_{sp2}). For linear elastic materials (E_{sp1}, E_r) and (E_{sp2}) are constants, but for non-linear elastic and elasto-plastic materials, (E_{sp1}, E_r) and (E_{sp2}) are functions of strain. The free strain due to shrinkage, temperature, etc, are denoted by (e_f), while the strain induced during the construction sequence, are denoted by (e_r). Hence, if $(u\ )$ and $(w\ )$ are assumed to exclude the displacements corresponding, to (e_f) and (e_r), the stresses in the layers are given by:

$$s_{sp1}=E_{sp1}(u_{sp1,x}-z_{sp1}.w_{sp1,xx}+e_{rsp1}-e_{fsp1}) \tag{50}$$

$$s_r=E_r(u_{r,x}-z_r.w_{r,xx}+e_{rr}-e_{fr}) \tag{51}$$

$$s_{sp2}=E_{sp2}(u_{sp2,x}-z_{sp2}.w_{sp2,xx}+e_{rsp2}-e_{fsp2}) \tag{52}$$

The axial forces, (F_{sp1}, F_r) and (F_{sp2}), and moments (M_{sp1}, M_r), and (M_{sp2}) are obtained by integrating the stresses, multiplying by the appropriate lever arms, (z_{sp1}, z_r) and

(z_{sp2}), in the case of moments over the cross section area of each layer denoted by (A_{sp1}, A_r) and (A_{sp2}). Hence:

$$F_{sp1} = \int s_{sp1}.dA_{sp1} \tag{53}$$
$$F_r = \int s_r.dA_r \tag{54}$$
$$F_{sp2} = \int s_{sp2}.dA_{sp2} \tag{55}$$
$$M_{sp1} = -\int s_{sp1}.z_{sp1}.dA_{sp1} \tag{56}$$
$$M_r = -\int s_r.z_r.dA_r \tag{57}$$
$$M_{sp2} = -\int s_{sp2}.z_{sp2}.dA_{sp2} \tag{58}$$

Substituting Eqs. (50) to (52) into Eq.s (53) to (58) gives:

$$F_{sp1} = \int E_{sp1}.(u_{sp1,x} - z_{sp1}.w_{sp1,xx} + e_{rsp1} - e_{fsp1})dA_{sp1} \tag{59}$$
$$F_r = \int E_r.(u_{r,x} - z_r.w_{r,xx} + e_{rr} - e_{fr})dA_r \tag{60}$$
$$F_{sp2} = \int E_{sp2}.(u_{sp2,x} - z_{sp2}.w_{sp2,xx} + e_{rsp2} - e_{fsp2})dA_{sp2} \tag{61}$$
$$M_{sp1} = -\int E_{sp1}.(u_{sp1,x} - z_{sp1}.w_{sp1,xx} + e_{rsp1} - e_{fsp1}).z_{sp1}.dA_{sp1} \tag{62}$$
$$M_r = -\int E_r.(u_{r,x} - z_r.w_{r,xx} + e_{rr} - e_{fr}).z_r.dA_r \tag{63}$$
$$M_{sp2} = -\int E_{sp2}.(u_{sp2,x} - z_{sp2}.w_{sp2,xx} + e_{rsp2} - e_{fsp2}).z_{sp2}.dA_{sp2} \tag{64}$$

IF (E_{sp1}, E_r), and (E_{sp2}) are constants, integration of Eqs. (59) to (64) gives:

$$F_{sp1} = E_{sp1}.A_{sp1}.u_{sp1,x} + E_{sp1}.(\bar{e}_{rsp1} - \bar{e}_{fsp1}) \tag{65}$$
$$F_r = E_r.A_r.u_{r,x} + E_r.(\bar{e}_{rr} - \bar{e}_{fr}) \tag{66}$$
$$F_{sp2} = E_{sp2}.A_{sp2}.u_{sp2,x} + E_{sp2}.(\bar{e}_{rsp2} - \bar{e}_{fsp2}) \tag{67}$$
$$M_{sp1} = E_{sp1}.I_{sp1}.w_{sp1,xx} \tag{68}$$
$$M_r = E_r.I_r.w_{r,xx} \tag{69}$$
$$M_{sp2} = E_{sp2}.I_{sp2}.w_{sp2,xx} \tag{70}$$

where I_{sp1}, I_r and I_{sp2} are the second moment of area of upper steel plate , steel bars and lower steel plate respectively and $\bar{e}_r$ is the integral of the strain function over the cross-section area of the corresponding material. After substituting Eqs. (65) to (70) into the basic equilibrium and compatibility Eqs. (7), (8) ,(12) , (25), (26) and (46) gives:

$$E_{sp1}.I_{sp1}.w_{sp1,xxxx} + E_r.I_r.w_{r,xxxx} + E_{sp2}.I_{sp2}.w_{sp2,xxxx} - h_1[E_r.A_r.u_{r,xxx} + E_r(\bar{e}_{rr} - \bar{e}_{fr})_{,xx} + (h_1 + h_2).[E_{sp2}.A_{sp2}.u_{sp2,xxx} + E_{sp2}.(\bar{e}_{rsp2} - \bar{e}_{fsp2})_{,xx} = r \tag{71}$$

$$E_{sp1}.I_{sp1}.w_{sp1,xxxx}+E_r.I_r.w_{r,xxxx}+E_{sp2}.I_{sp2}.w_{sp2,xxxx}+h_1[E_{sp1}.A_{sp1}.u_{sp1,xxx}+ E_{sp1}(\bar{e}_{rsp1}-\bar{e}_{fsp1})_{,xx}]-h_2.[E_{sp2}.A_{sp2}.u_{sp2,xxx}-E_{sp2}.(\bar{e}_{rsp2}-\bar{e}_{fsp2})_{,xx}=r \tag{72}$$

$$E_{sp1}.A_{sp1}.u_{sp1,xx}+E_{sp1}.(\bar{e}_{rsp1}-\bar{e}_{fsp1})_{,x}+E_r.A_r.u_{r,xx}+E_r(\bar{e}_{rr}-\bar{e}_{fr})_{,x} +E_{sp2}.A_{sp2}.u_{sp2,xx}+E_{sp2}.(\bar{e}_{rsp2}-\bar{e}_{fsp2})_{,x}=0 \tag{73}$$

$$E_{sp1}.A_{sp1}.u_{sp1,xx}+E_{sp1}.(\bar{e}_{rsp1}-\bar{e}_{fsp1})_{,x}-k_{s1} [(u_{sp1}-z_{sp1i}.w_{sp1,x})-(u_{sp1}-z_{sp1i}.w_{sp1,x})]=0 \tag{74}$$

$$E_r.A_r.u_{r,xx}+E_r.(\bar{e}_{rr}-\bar{e}_{fr})_{,x}+k_{s1}[(u_{sp1}-z_{sp1i}.w_{sp1,x})-(u_r-z_r.w_{r,.x})] -k_{s2}[(u_r-z_{ri}.w_{r,x})-(u_{sp2}-z_{sp2i}.w_{sp2,x})]=0 \tag{75}$$

$$E_r.I_r.w_{r,xxxx}-z_{ri}.[E_{sp2}.A_{sp2}.u_{sp2,xxx}+E_{sp2}.(\bar{e}_{rsp2}-\bar{e}_{fsp2})_{,xx}]+ z_{ri}.[(E_{sp1}.A_{sp1}.u_{sp1}+E_{sp1}(\bar{e}_{rsp1}-\bar{e}_{fsp1})_{,xx} -k_{n2}.(w_{sp2}-w_r)+k_{n1}(w_r-w_{sp1})=r_b \tag{76}$$

4. NUMERICAL SOLUTION

Eqs. (71) o (76) contain derivatives of third order in (u) and fourth order in (w), which can be expressed in finite (central) difference form using five node points, for example, the derivatives of (w) at node (n) can be expressed as:

$$w_{n,x}=\frac{w_{n+1}-w_{n-1}}{2.\Delta x} \tag{77}$$

$$w_{n,xx}=\frac{w_{n+1}-2.w_n+w_{n-1}}{\Delta x^2} \tag{78}$$

$$w_{n,xxx}=\frac{w_{n+2}-2.w_{n+1}+2.w_{n-1}-w_{n-2}}{2.\Delta x^3} \tag{79}$$

$$w_{n,xxxx}=\frac{w_{n+2}-4.w_{n+1}+6.w_n-4.w_{n-1}+w_{n-2}}{\Delta x^4} \tag{80}$$

After expressing Eqs. (71) to (76) in finite difference form, the complete solution system of algebraic equations, six degrees of freedom per node, can be solved for the unknown displacements at the nodes, and it required two external nodes at each end of the beam. In general, since the model is done for uniform distribution load and to specify the boundary conditions, the point load P can be idealized as a uniform distribution load $r = P/\Delta x$, applied over single node spacing.

5. BOUNDARY CONDITIONS

Solution of the resulting set of algebraic equations requires the specification of boundary conditions. In general, the equations contain a derivative of fourth order thus it required two external nodes to specify the boundary conditions at each end. However, if each external node is assigned six degree of freedom per node, then twelve boundary conditions are required for each end of the beam and must be specified.

$w_c = 0$ at $x = 0$ when $x = L$ (81)

$w_{a,xx} = 0$ at $x = 0$ when $x = L$ (82)

$w_{b,xx} = 0$ at $x = 0$ when $x = L$ (83)

$w_{c,xx} = 0$ at $x = 0$ when $x = L$ (84)

$u_c = 0$ at $x = 0$ (85)

$u_{c,x} = 0$ at $x = L$ (86)

$u_{a,x} = 0$ at $x = 0$ when $x = L$ (87)

$u_{b,x} = 0$ at $x = 0$ when $x = L$ (88)

$V_a + V_b + V_c = R_r$ at $x = 0$ (89)

$V_a + V_b + V_c = R_l$ at $x = L$ (90)

$u_{a,xxxx} = 0$ at $x = 0$ when $x = L$ (91)

$u_{b,xxxx} = 0$ at $x = 0$ when $x = L$ (92)

$u_{c,xxxx} = 0$ at $x = 0$ when $x = L$ (93)

$U_{ab,x} = 0$ at $x = 0$ when $x = L$ (94)

Eq. (89) and (90) express the conditions that the sum of the shear forces in the layers are equal to the support reaction R_r and R_l.

It is noted that the free strain due to shrinkage and temperature, etc and strain induced during construction sequence are neglected .

6. COMPARISON WITH PREVOUS EXPERIMENTAL STUDIES

A computer program was written in FORTRAN language for the present model to solve the six basic differential equations. The main parameters affecting the behavior of any composite structure is slip between layers, so the convergence in the built program is controlled by slip which is taken as 0.001. The results obtained from this program were compared with other results obtained from experimental researches. Experimentally , few works on the continuous composite beams are found in literature as difficulties may arise during the testing of such beams and due to the high cost of the preparation and construction of test beams. Teraskiewicz tested two quarter-scale continuous composite beams as reported by Yam and Chapman [13]; they compare the results obtained by their numerical solution with that obtained by Teraskiewicz's tests, which show good agreement with the experimental values at different load levels.

7. YAM AND CHAPMAN’S EXAMPLE

A single continuous beam of two equal spans (336 cm) is subjected to concentrated load of (74.2) kN at the middle of each span, Fig.(2). Due to symmetry , half of the continuous beam is considered . the applied load is about (57 %) from the calculated ultimate capacity of the beam, so that the behavior of the beam is within the elastic range. The material properties of the beam are given in Table (1). All dimensions in the original reference are in imperial unit and have been presented here in SI-unit system . Table (2) and Figure(4) shows the max. slip for analytical and previous study.

Superscript (*) indicates assumed information as they are missing from the reference . Fig. (4) shows the same beam after strengthening by upper steel plate attached to the concrete slab (assuming that the steel plate has the same width of the concrete slab) by using " shear connectors ". The material property of steel plate is assumed to be the same as that for steel beam.

8. CONCLUSIONS

Multi-layer composite continuous beams are very important element used in different types of construction, the present model deal with three layers. The present model lead to the following conclusions :

1. Composite multi-layered beam is relatively a new construction and can be used in many industries.
2. Also, it can be used for strengthening a damaged or weaken construction.
3. The main problem is the relative movement between layers which is handled and discussed in many researches.
4. The theory developed in the present study can be used in other branches of engineering specially in mechanical engineering since the material properties and types of connectors are not specified and the shear stiffness is assumed to be continuous over the whole beam.
5. A theory of three layer composite continuous beams based on Roberts' approach led to six differential equations and a computer program to solve these equations is presented in this paper.
6. A comparison with previous researches has been carried out to investigate the validity of the analysis . The comparison gives a close agreement.

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NOTATION

sp1, r, and sp2= Subscript denotes different layers , upper steel plate, steel bars and lower steel plate respectively.

A = Cross-sectional area of different layers.

h_1 = Distance between the centroids of upper steel plate to the steel bars.

h_2 =Distance between the centroids of steel bars to lower steel plate.

E_i = Modulus of elasticity of any layer.

F_i =The axial forces in layer (i).

I_i =Second moment of area for the layer (i).

k_{s1} and k_{s2} =Shear stiffness of the joint per unit length between successive layers.

k_{n1} and k_{n2} =Normal stiffness of the joint per unit length between successive layers.

L = span length.

M= External applied moment.

M_i =Moment for layer (i) .

P_1 and P_2 =Normal force per unit length at the upper and lower interface.

r_i =Live load.

r =Live load and dead load.

r = Distributed self-weight of layer (i).

R_r , R_l =Reaction at the right and the left supports.

U_{ij} = Slip between layer (i) and (j).

u_i =Displacements of the layer (i) in the x -direction.

W= Point load.

w_i =Displacements of the layer (i) in the z -direction.

w_{ij} =Separation at the interface between the layer (i) and (j).

x.= Subscript denote differentiation.

z_{ai} =Z-coordinate of interface relative to local x-z axes in layers a, b and c.

e_f =Free strain due to shrinkage, temperature etc.

e_r = Strain induced during the construction sequence.

$\bar{e}$ =Integration of strain function over cross section area of the material.

e =Strain in any layers .

s =Stress in any layers.

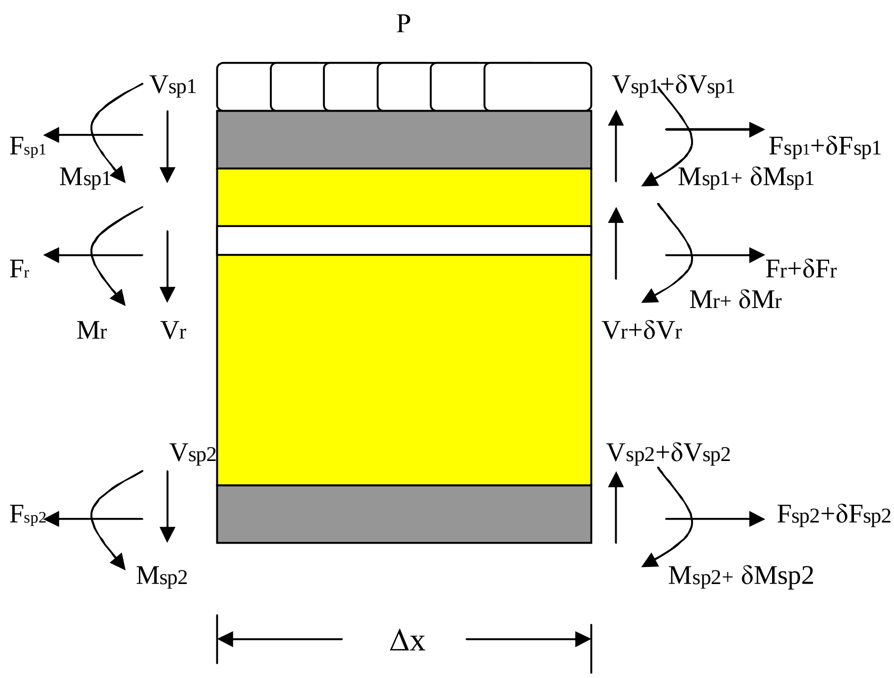
Δx =Spacing between nodes.

Table (1) The material properties of the beam

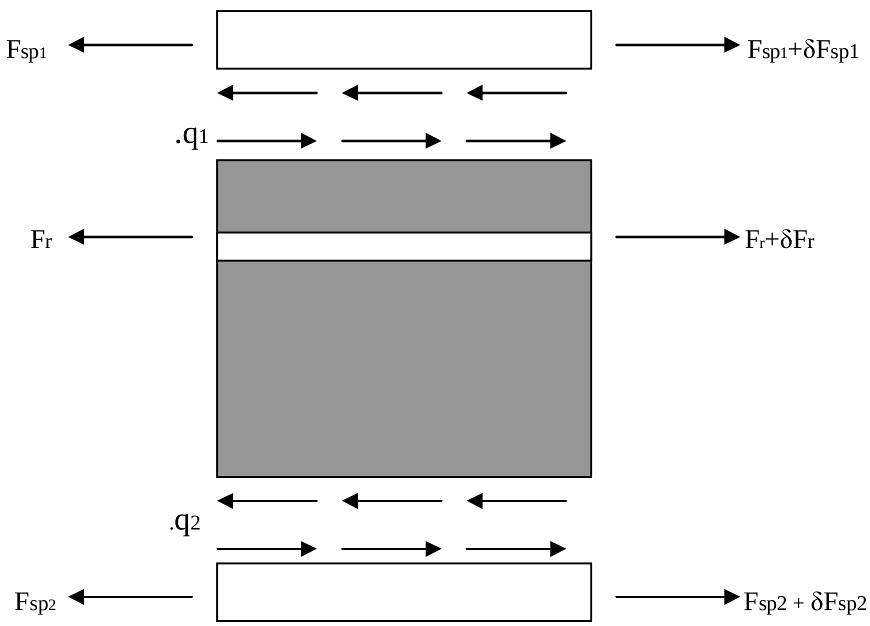
No.	Material	property	value
1	Concrete	Modulus of elasticity Ec N/mm^2	21000
2	Steel plate Steel bar Steel reinforcement	Modulus of elasticity Es N/mm^2	210000
3	Shear connectors	Connector modulus N/mm^2 Stud connector(dia. *height)mm Spacing (mm) No. of bars/row	1000 16*100 250 1

Table (2) Comparison between the numerical solution and Yam's example

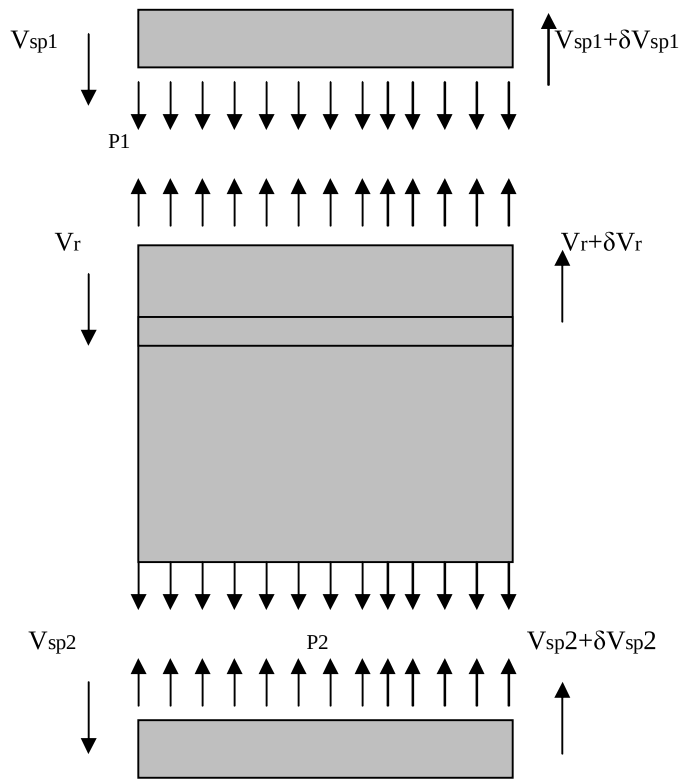
No.	Method of solution	Numerical solution			Yam's example
1	No. of nodes	10	15	20	
2	Max. slip (mm)	0.88	0.94	0.97	0.97



a. element of a three layers composite beam

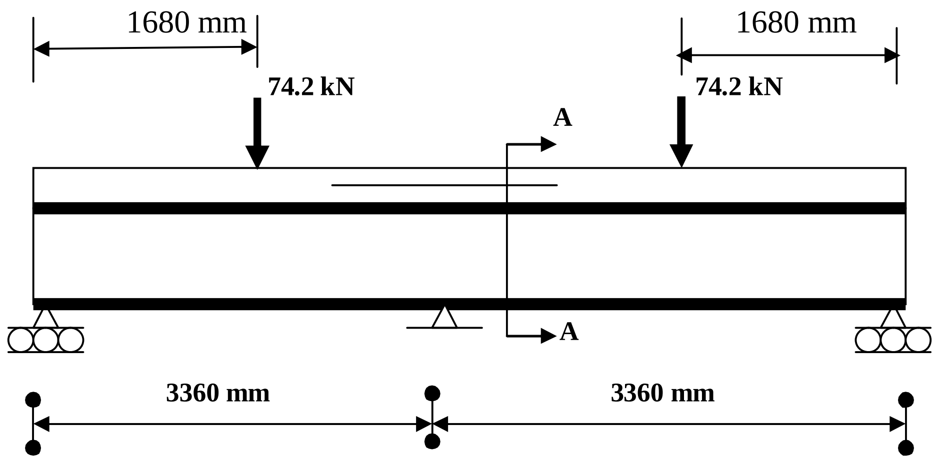


b. Slip

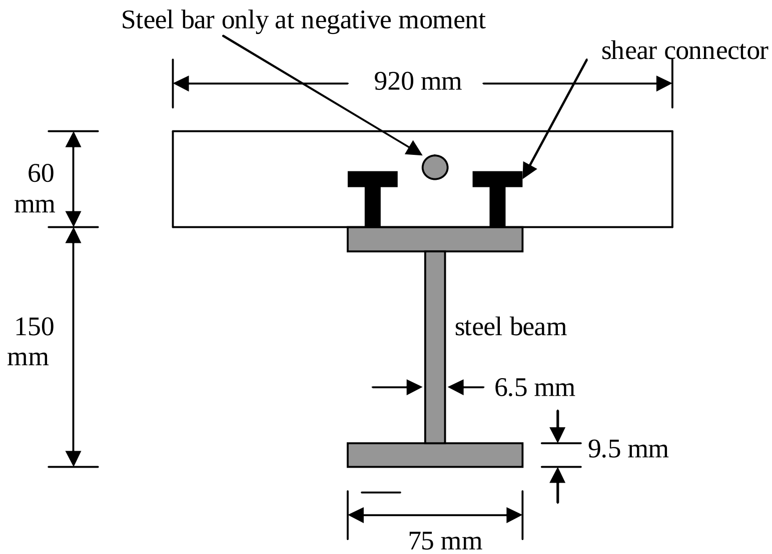


c. Separation

Figure (1): Composite three layers element
(a) cross-section (b) slip (c) separation

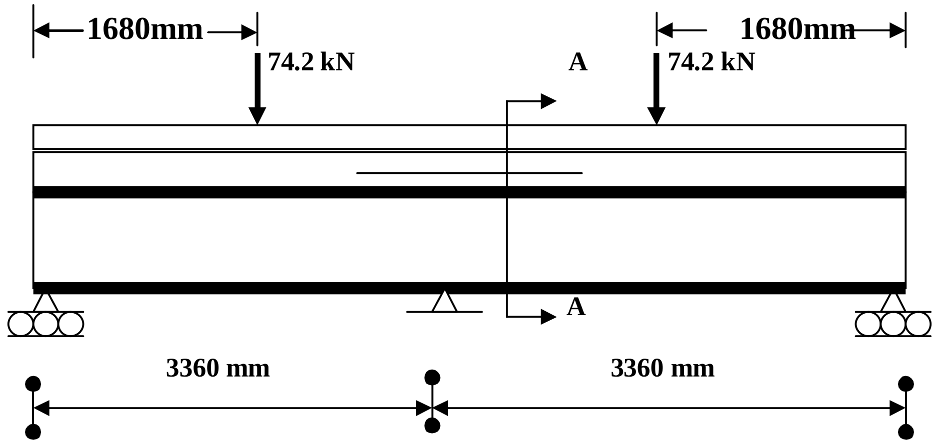


(a) continuous composite beam before strengthening

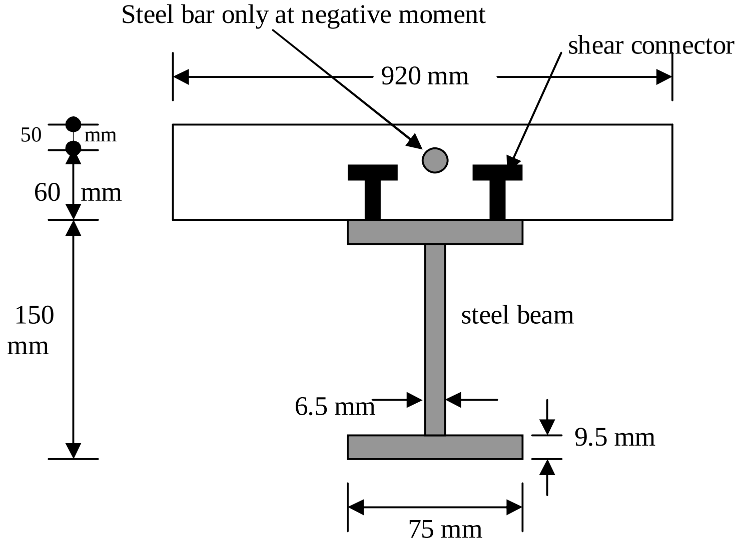


(b) Section A-A at the beam

Figure (2): (a) continuous composite beam before strengthening
(b) Section a-a at the beam



(a) continuous composite beam after strengthening



(b) Section A-A at the beam

Figure (3): (a) continuous composite beam after strengthening
(b) Section A-A at the beam

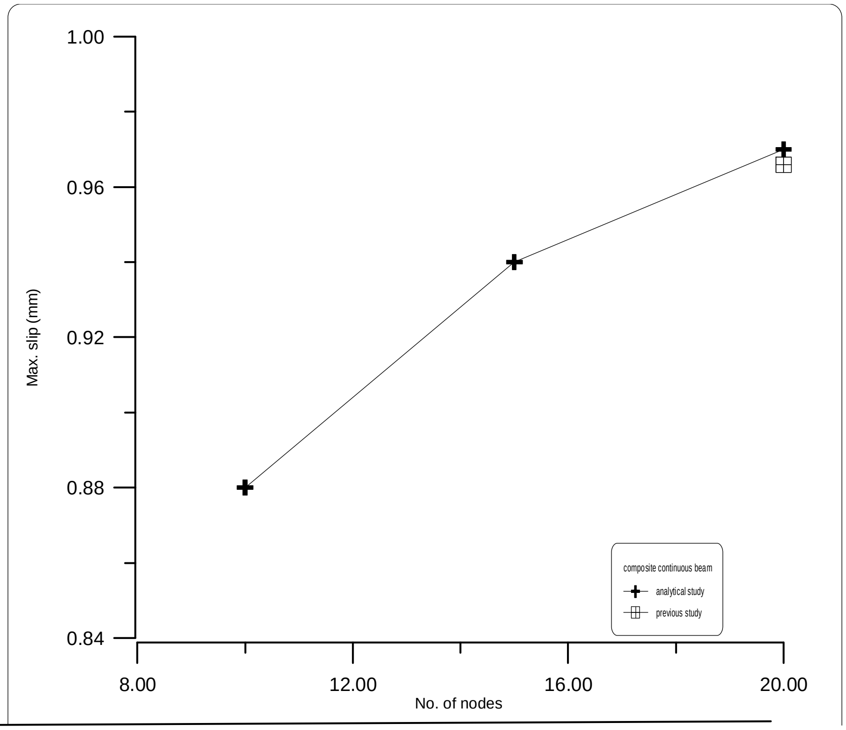


Figure (4): Comparison between analytical and previous study

تصرف العتبات المركبة متعددة الطبقات المستمرة ذات التداخل الجزئي

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الخلاصة

تعتبر المنشآت ذات المقاطع المركبة ذات أهمية كبيرة في الإنشاءات الحديثة ودخلت في الكثير من الصناعات البحرية وصناعة الطائرات ، في هذه الدراسة تم الاعتماد بصورة رئيسية على الطريقة التي اعتمدها العالم (روبرت) والخاصة بتحليل عتبات بسيطة الإسناد ذات مقاطع مركبة من مادتين والتي تم ربطهما بواسطة رباطات القص . هذا التحليل شمل العتبات المستمرة ذات ثلاثة مقاطع مختلفة من حيث الأبعاد وخواص المواد المكونة لها .إن المشكلة الرئيسية لمثل هذه المقاطع الإنشائية هي الإزاحة الأفقية بين الطبقات والتي لا يمكن منعها بصورة نهائية حتى مع زيادة رباطات القص . للعتبات المستمرة ذات مقاطع متكونة من ثلاثة مواد هناك إزاحات أفقية وعمودية بين الطبقات يجب أخذها بنظر الاعتبار ، اعتمد التحليل على معادلات التوازن والتوافق حيث كانت نتيجة التحليل الوصول الى مجموعة من المعادلات يبلغ عددها ست معادلات يمكن حلها بواسطة طريقة الفروق المحددة . النموذج الذي تم تحليله عبارة عن عتبة مستمرة ذات مقطع مكون من ثلاثة مواد ، طبقة وسطى من الخرسانة فوقها وتحتها طبقة من مادة فولاذية . نظرا" لقلة البحوث المتعلقة بفحص العتبات المستمرة تمت المقارنة مع النموذج الذي تم فحصه من قبل العالم (يام) وبينت النتائج تقاربا" من حيث الازاحات الأفقية .